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SOME NONINTERACTION PROPERTIES OF LINEAR MULTIVARIABLE CONTROL SYSTEMS

A THESIS

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SOME NONINTERACTION PROPERTIES OF

LINEAR MULTIVARIABLE CONTROL SYSTEMS

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TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

LIST OF ILLUSTRATIONS

Figure | Page

SUMMARY

This dissertation investigates noninteraction properties of linear time-invariant multivariable control systems. Three types of noninteraction properties are considered: selective output invariance, decoupling, and minimal input-output interaction. In many operational situations, it is necessary to know the noninteraction properties of a system in order to simplify the task of controlling it.

Simply stated, the term "selective output invariance" connotes the property whereby a specified input component has no effect at all on a specified output component. The system is said to be "decoupled" if a single input influences a single output, provided that the system has the same number of input and output components. The property of "minimal input-output interaction" in turn be related to a system transfer function which is output controllable and has a maximum number of zero elements (i.e., zero rational functions of s).

In designing a system, a feedforward compensator and a state feedback compensator are introduced to obtain a system with a desired noninteraction property. In this dissertation, methods are developed to compensate the system in order that it will exhibit any of the three noninteraction properties according to which of the appropriate compensators have been selected.

The decoupling problem has been investigated extensively during the last decade. Although necessary and sufficient conditions

for decoupling are known (see Falb and Wolovich [7]), a methodological development is adopted in order to provide a different version of conditions for decoupling. The notion of "row coefficient matrix (RCM)" is introduced and plays an important role in giving an intuitive explanation of the solution procedure for the decoupling problem.

The solution of the selective output invariance problem serves as a foundation for solution of the other noninteraction problems. That is to say, the properties of decoupling and minimal input-output interaction are approached through an analysis that is based on results of the selective output invariance problem. The minimal input-output interaction problem is, conceptually, an extension of the decoupling problem, but, because of some technical methodological difficulties, this dissertation considers the problems to be independent of each other.

The major result of the investigation is the development of methodologies for studying the selective output invariance problem, the decoupling problem, and the minimal input-output interaction problem. Sufficient conditions for selective output invariance are developed, and both necessary and sufficient conditions for decoupling are developed and compared with the result of Falb and Wolovich. Although a general solution to the minimal input-output interaction problem is not obtained, two special cases are investigated and solved.

As an extension of the research topic, some other properties such as stabilization, parameter insensitivity, and functional reproducibility of the decoupled system are also examined.

CHAPTER I

INTRODUCTION

1.1 Problem Statement

The purpose of this dissertation is to develop a methodology for the design of linear time-invariant multivariable control systems in such a way that the compensated systems will exhibit some noninteraction properties among specified input and output components. Noninteraction properties have been playing important roles in the analysis and design problems of linear multivariable control systems. In the last decade, special attention has been focused on the so-called decoupling problem. A system is decoupled if a single input influences only a single output. This problem arises in many different contexts and has been solved by many different mathematical approaches. Falb and Wolovich [7] were the first to establish necessary and sufficient conditions for decoupling; Wonham and Morse [35] solved the decoupling problem using a geometrical approach under several different assumptions about system characteristics; and Sato and Lopresti [30] extended the decoupling problem to include the case where a subset of the "output set" (the set of all outputs from the system) is the candidate for decoupling. This dissertation is aimed primarily at defining several types of noninteraction properties, and at developing a design methodology so that the system exhibits desired noninteraction properties.

The system of interest is defined as

$$\dot{x}(t) = A\ x(t) + B\ u(t) \tag{1.1}$$

$$y(t) = C\ x(t)$$

where x is an n-state vector, y an m-output vector, u an r-input vector (such that m, r $\leq$ n), and A, B, C are (n x n), (n x r) and (m x n) constant matrices, respectively. By introducing a control law of the form

$$u(t) = F\ x(t) + G\ v(t) \tag{1.2}$$

where v is a new r-input vector, F an (r x n) matrix, and G an (r x r) matrix, the following system results:

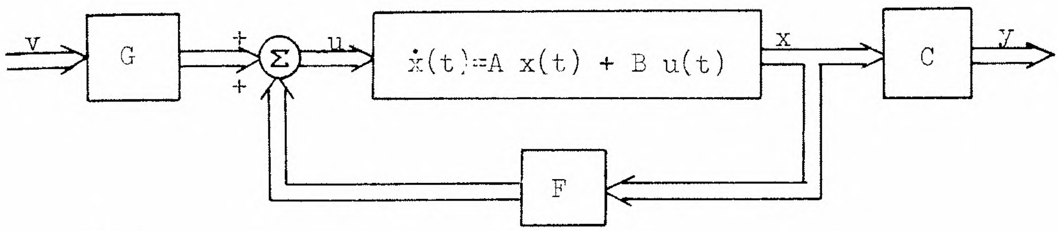
$$\dot{x}(t) = (A + BF)\ x(t) + BG\ v(t) \tag{1.3}$$

$$y(t) = C\ x(t).$$

The transfer function matrix W(s) of the system (1.3) is given by

$$W(s) = C(sI - A - BF)^{-1}BG \tag{1.4}$$

where W(s) is an (m x r) matrix, s being the Laplace transform variable. The configuration of the system (1.3) with the control law of equation (1.2) is illustrated by the following diagram:



Notice that if the matrix G is substituted by an identity matrix and the matrix F by a zero matrix in equation (1.2), then

$$u(t) = v(t). \tag{1.5}$$

Thus, the structure of the system (1.3) turns out to be identical with the original system (1.1). The transfer function matrix of the system (1.1) is given by

$$W(s) = C(sI - A)^{-1}B. \tag{1.6}$$

For the system (1.3), this dissertation will consider the following three types of noninteraction properties: selective output invariance, decoupling, and minimal input-output interaction.

The selective output invariance property was initially introduced by Zunde [37], who defined it as follows:

<u>Definition</u>

Let T be a set of time at which the behavior of the system is defined, and let t_0, $t_1 \varepsilon T$ and $t_0 < t_1$. The i-th component $y_i(t)$ of the output vector $y(t)$ is selectively output invariant with respect to the j-th component $v_j(t)$ of the input vector $v(t)$ at time t_0, if there exists $t_1 > t_0$ such that the values of $y_i(t)$ on the time interval $[t_0,t_1]$ do not depend on any of the values of $v_j(t)$ on $[t_0,t_1]$.

It can be shown (see Zunde [37]) that if a linear time-invariant system is selectively i-th output invariant with respect to the j-th input for some time $t_0 \varepsilon T$, then it is selectively invariant for all $t_0 \varepsilon T$ and all intervals $[t_0,t_1]$ where t_0, $t_1 \varepsilon T$; and that when the linear time-invariant system is described by a transfer function matrix $W(s)$, the system is selectively i-th output invariant with respect to the j-th input if and only if

$$w_{ij}(s) = 0 \tag{1.7}$$

where $w_{ij}(s)$ is the (i,j)-th element of $W(s)$ and 0 represents a zero rational function of s. The selective output invariance is defined as follows:

<u>Definition</u>

The system (1.3) is selectively i-th output invariant with respect to the j-th input if and only if the (i,j)-th element of its transfer function matrix is equal to zero.

Physically, when the system is selectively i-th output invariant with respect to the j-th input, then the j-th input has no effect at all on the i-th output, assuming zero initial conditions.

An extension of the selective output invariance property leads to the definition of decoupling. It is implicitly assumed in discussing the decoupling property that $m = r$, that is, that the number of output components of the system is equal to the number of input components. Roughly speaking, the system is said to be decoupled if a single input influences a single output. For the linear time-invariant system with the transfer function matrix $W(s)$, we can simply extend the description of the selective output invariance to state the following decoupling property: The system is decoupled if each row and each column of $W(s)$ contain only one nonzero element. Then the definition of decoupling can be stated in a simple form in terms of equation (1.4).

Definition

The system (1.3) is said to be decoupled if one of the following two conditions is satisfied: (1) The system transfer function matrix $W(s)$ is diagonal and diagonal elements are nonzero rational functions of s; (2) $W(s)$ can be transformed into a diagonal form, whose diagonal elements are nonzero rational functions of s, by an appropriate permutation of its rows and columns.

Since, without loss of generality, the positions of the output components in the output vector $y(t)$ can be adjusted by exchanging the rows of the matrix C before performing the system design, and the positions of the input components can be adjusted by exchanging the columns of the matrix B, only the first statement of this definition will be examined for decoupling.

The third type of noninteraction property is minimal input-output interaction, and is defined as follows:

Definition

Let S_1 and S_2 be two linear constant coefficient output controllable systems each with r inputs and m outputs. Let n_1 and n_2 be the number of nonzero elements in the transfer function matrices $W_1(s)$ and $W_2(s)$ of S_1 and S_2, respectively. Then, the system S_1 is said to be less input-output interacting than the system S_2 if and only if $n_1 < n_2$. Let M be the class of all output controllable linear constant coefficient systems with r inputs and m outputs. If for some system $S \epsilon M$ there is no system $S' \epsilon M$ such that S' is less input-output interacting than S, we shall say that the system S is a minimally input-output interacting system in the class M. In general, S is not unique.

From the definitions of decoupling and minimal input-output interaction, it is immediately clear that a decoupled system is always minimally input-output interacting.

One of the objectives of this dissertation is to investigate the decoupling property, and in particular, the selection of the appropriate matrices G and F so that the system (1.3) can be decoupled. The selective output invariance property of the system (1.3) will be studied in the context of the decoupling problem. Again, selection of the appropriate matrices G and F to obtain desired output invariance properties in the system becomes a main objective of the discussion.

If the necessary and sufficient conditions set forth by Falb and Wolovich [7] do not hold for the system (1.3), then the system can not be decoupled. In such a case, we might be interested in the problem of feedback compensation which minimizes the degree of interaction among input and output components. Then for the system (1.3) the problem is to choose the elements of the matrices G and F so that the system is minimally input-output interacting. In other words, by the selection of appropriate matrices G and F, the transfer function matrix W(s) must be designed so as to include the maximum number of zero elements (zero rational functions of s) and to exhibit the output controllability property.

1.2 Examples of Applications

In many operational situations it is advantageous to know the noninteraction properties of the system. A few of these are listed here to indicate the range of the noninteraction properties encountered in practice.

One of the earliest examples of noninteracting control was the Boksenbom and Hood gas-turbine engine [1], for which the control of more than one variable was desired, i.e., control of all of the variables that may be caused by excessive temperature, speed, or torque. Controlling more than one variable generally introduces an interaction among the controlled variables, since a new setting for just one variable may cause changes in other variables. If these other variables are operating at or near the levels which produce a maximum output, this interaction may create excessive values and possibly result in damage to the engine (i.e., exceeding specified burner operating limits may cause burner blowout). It would therefore be desirable for each new setting of a controlled variable to affect only the variable being set, thus allowing separate noninteracting control of all the controlled variables.

Chein et al. [5] attempted to design a noninteracting boiler control in such a way that a change in the pressure setting point does not alter the water level and a change in the water level does not alter the boiler pressure. The designer had a chance to improve on the boiler itself, but at the same time the design of a decoupling controller became one of the topics for the study of structures and characteristics of multivariable control systems. Although the primary purpose of designing the decoupling controller was to simplify actual boiler operations, another advantage of decoupling is that it makes possible the simplification of the system analysis itself. That is, the design problem moves from the multivariable system domain to that of the well-known single-variable domain.

Other examples of the noninteracting control are found in the boiler control by means of commercially available PID-process controller [4], turbopropeller engine control [16], and the operation of a rotating dc-to-ac converter [27].

In 1967, Falb and Wolovich [7] presented necessary and sufficient conditions for decoupling, and observed that, in certain cases, a system can be decoupled (without extra equipment) simply by making sure that the controllers link the best combination of system inputs, states and outputs. A good illustration of the design of the decoupling property is given in the following example of a hydraulic system.

Suppose that the two-tank system of Figure 1 has two control inflows q_{i1} (in^3/sec) for the left tank and q_{i2} for the right tank, as well as two outflows q_{o1} and q_{o2}. R_1, R_2 and R are hydraulic

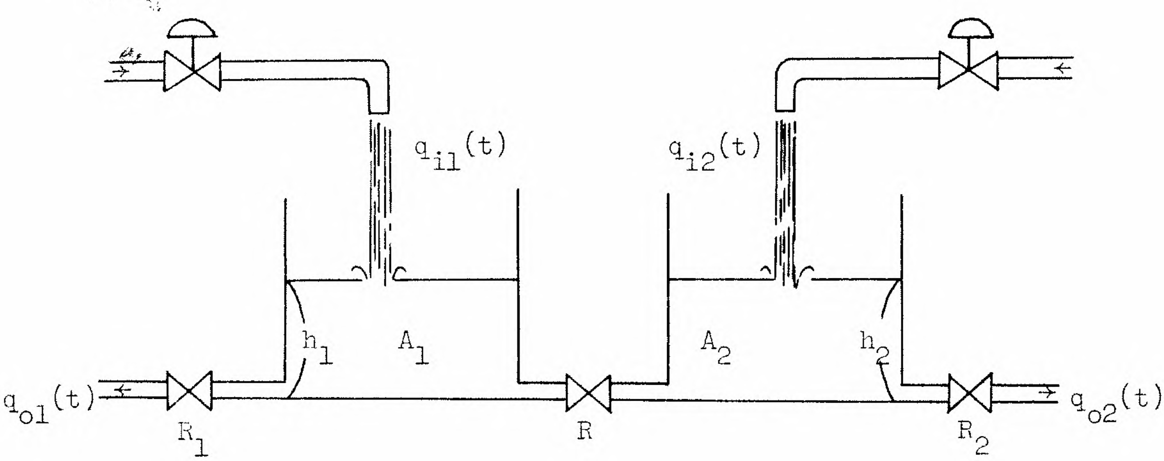


Figure 1. Two-Tank System.

resistances (sec/in^2), and A_1 and A_2 are cross-sectional areas (in^2) of the left and right tanks. The water levels of the tanks are denoted by h_1 and h_2 (in).

The inflows, outflows and the water levels in the tanks are related by the following system equations.

$$\begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{A_1R_1} - \frac{1}{A_1R} & \frac{1}{A_1R} \\ \frac{1}{A_2R} & -\frac{1}{A_2R_2} - \frac{1}{A_2R} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{A_1} & 0 \\ 0 & \frac{1}{A_2} \end{bmatrix} \begin{bmatrix} q_{i1} \\ q_{i2} \end{bmatrix} \tag{1.8}$$

$$\begin{bmatrix} q_{o1} \\ q_{o2} \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1} & 0 \\ 0 & \frac{1}{R_2} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \tag{1.9}$$

For the purpose of explanation, assume that $R_1 = R_2 = R = 1$ and $A_1 = A_2 = 1$. Then the system equations (1.8) and (1.9) are given by

$$\begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} q_{i1} \\ q_{i2} \end{bmatrix} \tag{1.10}$$

$$\begin{bmatrix} q_{o1} \\ q_{o2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \tag{1.11}$$

The transfer function matrix of this system is given as follows:

$$W(s) = C(sI - A)^{-1} = \frac{1}{s^2+4s+3}\begin{bmatrix} s+2 & 1 \\ 1 & s+2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s+2}{(s+1)(s+3)} & \frac{1}{(s+1)(s+3)} \\ \frac{1}{(s+1)(s+3)} & \frac{s+2}{(s+1)(s+3)} \end{bmatrix} \qquad (1.12)$$

From equation (1.12) we obtain

$$Q_{o1}(s) = \frac{s+2}{(s+1)(s+3)}\, Q_{i1}(s) + \frac{1}{(s+1)(s+3)}\, Q_{i2}(s)$$

$$Q_{o2}(s) = \frac{1}{(s+1)(s+3)}\, Q_{i1}(s) + \frac{s+2}{(s+1)(s+3)}\, Q_{i2}(s)$$

where $Q_{i1}(s)$, $Q_{i2}(s)$, $Q_{o1}(s)$, $Q_{o2}(s)$ denote the Laplace transforms of $q_{i1}(t)$, $q_{i_2}(t)$, $q_{o1}(t)$, $q_{o2}(t)$, respectively.

Consider now the compensated system shown in Figure 2. Two inputs are added to the original system. The information about water levels is fed back to the input terminals and the following amount of water is supplied to the tanks through pipe 1 and pipe 2. Suppose that

$$\bar{q}_{i1}(t) = 2\, h_1 - h_2, \qquad (1.13)$$

$$\bar{q}_{i2}(t) = - h_1 + 2h_2. \qquad (1.14)$$

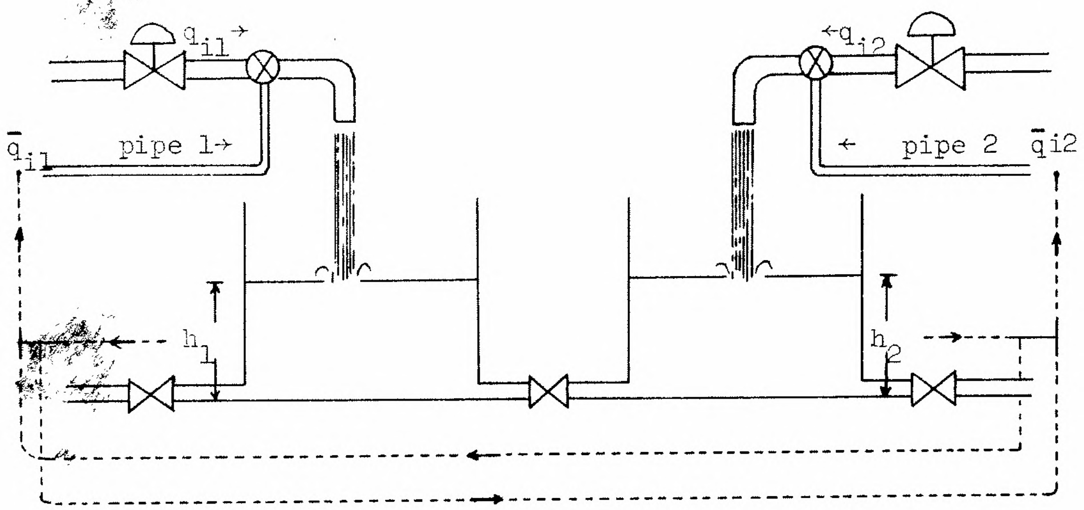


Figure 2. Compensated Two-Tank System.

If $\bar{q}_{i1}(t)$ or $\bar{q}_{i2}(t)$ takes negative value, such amount of water must be pumped out from pipe 1 or pipe 2.

The new system differential equations representing the system of Figure 2 are described by adding new input terms $\bar{q}_{i1}$ and $\bar{q}_{i2}$ to equation (1.8).

$$\begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} q_{i1} \\ q_{i2} \end{bmatrix} + \begin{bmatrix} \bar{q}_{i1} \\ \bar{q}_{i2} \end{bmatrix} \tag{1.15}$$

$$= \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} q_{i1} \\ q_{i2} \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

$$\begin{bmatrix} q_{o1} \\ q_{o2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \tag{1.16}$$

Then the transfer function matrix is computed and given as follows:

$$W(s) = \begin{bmatrix} \frac{1}{s} & 0 \\ 0 & \frac{1}{s} \end{bmatrix}$$

which implies that

$$Q_{o1}(s) = \frac{1}{s} Q_{i1}(s) \tag{1.17}$$

$$Q_{o2}(s) = \frac{1}{s} Q_{i2}(s) \tag{1.18}$$

We see that the system is decoupled.

In this example, the decoupling property of the two-tank system has been achieved by providing the feedback of the water levels in the tanks to the input terminals. As seen in equation (1.17) and equation (1.18), the outflow of each tank depends on the input to its own tank. Thus, as the result of decoupling, the two tanks seem to be working individually, even though the system structure of the two-tank system is expressed by the interrelated mathematical equations.

1.3 Literature Survey

There exists considerable body of control theory literature pertinent to the decoupling property by state variable feedback. Complete solutions, giving necessary and sufficient conditions for decoupling, can be found in Falb and Wolovich [7], Gilbert [11], Wonham and Morse [35]. In addition, Morse and Wonham [22] and Chen [6] have written useful reviews. Below, we offer a brief survey of this literature, and discuss some of the main contributions leading to the research effort described in present dissertation.

Although significant advances in the theory of decoupling have been made since 1963, certain classical contributions prior to that year can not be ignored, because some of those contributions pointed to operational difficulties of interacting systems in order to show the advantages of designing decoupled systems. The earliest known investigation of noninteraction was made as early as 1939 in the Soviet Union by Voznesenskii (see [22]), who was concerned with the control of power station turbines. It is known [27] that, in 1940, Luzin studied complex multiloop structures and formulated the conditions under which some output variable of a set of differential equations is independent of some forcing function. In the United States, Boksenbom and Hood [1] first developed a decoupling procedure for a jet engine controller in 1949. Thereafter, Chein et al [5], Chatterjee [4], Jeffrey [14], Mitchell and Webb [19], Kvanagh [16] also indicated the necessity of incorporating decoupling properties into real control systems. (Some of their results have already been introduced in the previous section). As far as theoretical developments prior to 1963 are concerned, there were few concrete results. Freeman [8] and Kavanagh [16] were applying transfer function matrix methods to the decoupling problem, but a general solution of the decoupling problem was not found.

Owing to the state space approach developed by Kalman [15] in 1960, Morgan [20] formulated the decoupling problem as a state feedback problem. Morgan presents a sufficient condition for decoupling and under this condition defines a rather restrictive class of control laws which decouple. In addition to these, he discusses the stabilizability of the decoupled system by two different methods, a state variable feedback method and a transfer function matrix method. Although the research effort described in the present dissertation has not adopted Morgan's methodology, it should be noted that his paper stimulated, and suggested a fruitful direction for many subsequent investigations (including the one described herein). In 1965, Rekasius [28] investigated Morgan's problem and extended some of the results.

In 1967. Falb and Wolovich [7] established the necessary and sufficient conditions for decoupling of the system (1.3). This paper develops necessary and sufficient conditions for decoupling, which are the first complete solutions to the decoupling problem for a significant class of linear systems. In addition to this result, Falb and Wolovich present a characterization of the class of feedback matrices which decouple a system, a determination of the number of closed-loop poles which can be specified while decoupling, and a synthesis procedure for obtaining a desired closed-loop pole configuration.

The paper proposes a precise mathematical definition of decoupling as follows: Let $d_1, d_2, \ldots, d_m$ be given by

$$d_i = \min \{j: \; C_i A^j B \neq 0, \; j = 0, 1, \ldots, n-1\}$$

or

$$d_i = n-1 \text{ if } \; C_i A^j B = 0 \text{ for all } j \tag{1.19}$$

where C_i denotes the i-th row of C.

<u>Definition</u>

The matrices G and F, with G nonsingular, decouple the system (1.3) if and only if (1) $\mathrm{tr}\,\{L^i(F,G)\Omega\} = \mathrm{tr}\,\{L^i(F,G)\Omega_i\}$ $i = 1, 2, \ldots, m$; and (2) $\mathrm{tr}\,\{L^i(F,G)\Omega\} \neq 0$ $i = 1, 2, \ldots m$ where $\mathrm{tr}\{\cdot\}$ denotes the trace operation and $L^i(F,G)$ is the $(n \times m)$ matrix given by

$$L^i(F,G) = \begin{bmatrix} C_1[(A+BF)^{n-1} - p_{n-1}(A+BF)^{n-2} \cdots - p_{d_{i+1}}(A+BF)^{d_i}]BG \\ \vdots \\ C_i[(A+BF)^{d_i+1} - p_{n-1}(A+BF)^{d_i}]BG \\ (0) \end{bmatrix}$$

in which (0) is a zero matrix of size consistent with $L^i(F,G)$, p_k are scalars, Ω is the (m x n) matrix given by $\Omega=[v, v^1, \ldots, v^{n-1}]$ where the superscripts of element v represent the order of the differentiation with respect to time t, and Ω_i is the (m x n) matrix with its i-th row identical to the i-th row of Ω and zeros everywhere else.

This definition, though precise, is somewhat difficult to understand and lacks the intuitive picture of inputs controlling the outputs independently. Equivalent but more intuitive definitions of decoupling are given in Gilbert [11] and Mufti [23].

In discussing the decoupling property, the number of input components is assumed to be equal to the number of output components, that is, m = r. The first theorem proposed by Falb and Wolovich is given as follows:

Theorem 1.1

Let B* be the (m x m) matrix given by

$$B^* = \begin{bmatrix} C_1 A^{d_1} B \\ C_2 A^{d_2} B \\ \vdots \\ C_m A^{d_m} B \end{bmatrix}$$

Then there is a pair of matrices G and F which decouples the system (1.3) if and only if det $B^* \neq 0$.

The paper further develops the characterization of the set of all pairs G and F which decouple the system (1.3) under the assumption that B^* is nonsingular. The second theorem for decoupling is given as follows:

Theorem 1.2

If the pair G and F decouples the system (1.3), then the rank of $Q^i(F)$ is one for all i; conversely, if the rank of $Q^i(F)$ is one for all i and if B^* is nonsingular, then the pair B^{*-1} and F decouples the system (1.3), where $Q^i(F)$ is the (n x m) matrix given by

$$Q^i(F) = \begin{bmatrix} C_i(A+BF)^{n-1}B \\ C_i(A+BF)^{n-2}B \\ \vdots \\ C_i(A+BF)^{d_i}B \\ (0) \end{bmatrix} \tag{1.20}$$

where (0) is a zero matrix consistent with the order of $Q^i(F)$.

The matrix $Q^i(F)$ given by equation (1.20) coincides with a part of a "row coefficient matrix" corresponding to the i-th row of the system transfer function matrix (1.4), which will be introduced in Chapter II of this dissertation.

Gilbert [11] points out that a drawback of the necessary and sufficient conditions derived by Falb and Wolovich [7] is their cumbersome algebraic form which makes the choice of G and F difficult when n is large.

Taking a different approach from that which Falb and Wolovich used in the decoupling problem, Gilbert remains completely in the Laplace domain throughout his discussion, and utilizes some basic matrix theory operations. First he presents a quite intuitive definition of decoupling, which is also adopted in the dissertation.

Definition

The system (1.3) is decoupled if $W(s) = C(sI - A - BF)^{-1} BG$ is diagonal and nonsingular.

Although it is not clear in this definition what Gilbert means by the word "nonsingular," it may be interpreted to imply that the diagonal elements of $W(s)$ possess the nonzero rational functions of s. Gilbert obtains, by different ways, Theorem 1.1 of Falb and Wolovich; whereas Falb and Wolovich prove Theorem 1.1 by manipulating some algebraic expressions, Gilbert proves it more simply by introducing the property of F-invariance. The F-invariance is defined as a property of the system (1.3) which for any fixed G does not depend on F. This property will be discussed in Chapter III of the dissertation.

Wonham and Morse [35] generalized Gilbert's approach [11] in formulating the decoupling problem in a vector space setting. They introduce the concept of controllability subspace in order to establish a precise definition of decoupling in algebraic terms. Below, capital letters A, B, C, . . . denote matrices and tilded letters $\tilde{A}$,

$\tilde{\mathcal{B}}$, $\tilde{\mathcal{C}}$, . . . denote linear vector spaces. If K is a matrix, $\{K\}$ or $\tilde{K}$ represents the range of K. $\tilde{E}^n$ is real n-space. The controllable subspace of the system $\dot{x}(t) = A\,x(t) + Bu(t)$, denoted by $\{A \mid B\}$, is defined as

$$\{A \mid \tilde{B}\} = \tilde{B} + A\,\tilde{B} + \ .\ .\ . + A^{n-1}\,\tilde{B}.$$

With regard to system (1.3), suppose that a subspace $\tilde{R} \subset \tilde{E}^n$ is selected and that it is desired to find a conditions for the system such that $\tilde{R}$ is completely controllable. From the definition of the controllable subspace, we require

$$\{A + BF \mid \ \{BG\}\} = \tilde{R}.$$

This concept of controllable subspace is used to formulate the decoupling problem.

Thus, let us consider the i-th output element described by

$$y_i = C_i\ x$$

where C_i is of dimension (1 x n). (Wonham and Morse actually separate the matrix C into k partitioned matrices so that the vectors y_i corresponding to the i-th partitioned matrices C_i are regarded as physically significant groups of scalar output variables.) Describe the control law of equation (1.2) in the form of

$$u = Fx + \sum_{i=1}^{r} g_i v_i$$

where v_i is the i-th component of the input vector v and g_i is the i-th column of the matrix G. For v_i to control y_i completely, we must have

$$C_i\{A + BF \mid \{Bg_i\}\} = \mathcal{C}_i \tag{1.21}$$

where $\mathcal{C}_i$ is the range of C_i. Since the i-th control v_i is to leave the outputs y_j, $j \neq i$, unaffected, we require also

$$C_j\{A + BF \mid \{Bg_i\}\} = \{\underline{0}\} \quad j \neq i, \tag{1.22}$$

where $\{\underline{0}\}$ is the zero subspace. The expressions equation (1.21) and equation (1.22) turn out to be the definition of decoupling by Wonham and Morse.

Using certain algebraic properties, Wonham and Morse reduce equation (1.21) and equation (1.22) into more succinct forms, and then solve the decoupling problem for the following two different cases:

In the first case, the assumption is made that

$$\text{rank } C = n$$

which implies that there is a one-to-one mapping of state variables into output variables.

The second case has an assumption that

$$\text{rank } B = m.$$

This condition is equivalent to the assumption that the number of independent open-loop system inputs is exactly equal to the number of output components to be controlled.

The same authors extend their discussion in a subsequent article [22] in 1971, in which they present a third case related to Morgan's approach [20]. For the third case, the assumptions are described as

$$\text{rank } B = m,$$

$$\text{rank } C_i = 1 \qquad i = 1, 2, \ldots, m.$$

For each of the three cases, the solution of the decoupling problem is obtained by algebraic calculations in the vector space. Theorem 1.1 by Falb and Wolovich is, of course, proved as a solution of the third case.

Sato and Lopresti [30] generalized the decoupling problem to include the case where a subset of the "output set" (the set of all output components) is the candidate for decoupling. A system in which such decoupling is employed is referred to as "partially decoupled." The definition adopted by Sato and Lopresti is very similar to that of Falb and Wolovich and is given (using Falb and Wolovich's notation) as follows:

Definition

A subset $\{y_1, \ldots, y_p\}$ where $p \leq m$ of the output set of the system (1.3) is decoupled if for all $i = 1, 2, \ldots, p$ the following conditions hold, (1) $\mathrm{tr}\ \{L^i(F,G)\Omega\} = \mathrm{tr}\{L^i(F,G)\Omega_{j_i}\}$, $j_i \epsilon \{1, 2, \ldots, r\}$, (2) $\mathrm{tr}\ \{L^i(F,G)\Omega\} \neq 0$, (3) $\mathrm{tr}\ \{L^k(F,G)\Omega_{j_i}\} = 0$, $k \epsilon \{1, 2, \ldots, m\}$, $k \neq i$.

Notice that if $m = p = r$, the partial decoupling is equivalent to the definition of Falb and Wolovich.

Simply stated, Sato and Lopresti's aim in proposing partial decoupling is to compensate the system in such a way that the system transfer function matrix has a special structure where only p selected elements out of m outputs are decoupled. This is shown in the following matrix expression including four partitioned matrices:

$$W(s) = C(sI - A - BF)^{-1}BG = \begin{bmatrix} W_{11}(s) & W_{12}(s) \\ W_{21}(s) & W_{22}(s) \end{bmatrix}$$

where $W_{11}(s)$ is a $(p \times p)$ decoupled matrix, $W_{12}(s)$ and $W_{21}(s)$ are $(p \times (r-p))$

and $((m-p) \times p)$ zero matrices, respectively, and $W_{22}(s)$ is an $((m-p) \times (r-p))$ matrix for which the selection of the elements are not specified. They may or may not be zero elements.

The necessary and sufficient conditions for partial decoupling by Sato and Lopresti [30] are developed in an algorithm originally introduced by Silverman's work [31] on system invertibility. The main theorem of Sato and Lopresti is given as follows:

<u>Theorem 1.3</u>

The output subset $\{y_1, y_2, \ldots, y_p\}$ of the system (1.3) can be partially decoupled if and only if (1) rank $B_p^* = p$ and (2) rank D_α = rank D_α^a + rank D_α^b where

$$B_p^* = \begin{bmatrix} C_1 A^{d_1} B \\ \vdots \\ C_p A^{d_p} B \end{bmatrix},$$

and D_α, D_α^a, D_α^b are determined by the fairly complicated recursive calculations (given in [30]).

It will be shown in this dissertation that the first condition of Theorem 1.3 becomes the necessary and sufficient condition of "selective decoupling," which will be defined in Chapter IV.

Sato and Lopresti also answer the question which sets of inputs and outputs can be partially decoupled. It is shown that, for every linear time-invariant system, there exists a unique maximum set of outputs that can be partially decoupled. A procedure for constructing the maximum output set is also given.

In a new research direction which has occurred since 1971, the decoupling problem has been extended for different system structures. Wang [33] and Howze [13] deal with the linear multivariable systems with output feedback and obtain necessary and sufficient condition for decoupling. Sankaran and Srinath [29] consider stochastic decoupling and present the necessary and sufficient condition for decoupling in linear multivariable systems with known Gaussian noises. To a special class of nonlinear time-varying systems, a sufficient condition for decoupling is developed by Nazar and Rekasius [28].

The analysis employed in this dissertation is performed completely in the Laplace domain. However, the present work on the decoupling problem will differ from that of Gilbert [11] in that the approach is to first reduce the system transfer function matrix described by equation (1.4) into a simple system representation in terms of some "row coefficient matrices," rather than to attack the system (1.4) directly. The discussion will proceed in a systematic manner using some aspects of matrix theory and some fundamental properties of linear space theory. The solution procedure to decoupling problem will also provide the answer to the selective output invariance problem, which has never been considered as a related topic to the decoupling property. Zunde [37] introduced the definition of selective output invariance in 1968 in his research on system controllability. Based on Zunde's results, this dissertation will try to investigate further the selective output invariance problem for the system (1.3).

1.4 Organization of the Dissertation

Chapter II will introduce the notion of row coefficient matrix, and the system (1.3) will be represented in terms of these matrices. To provide an application of this new representation of the system (1.3), the output controllability criterion of the system will be examined. It will be shown that the system (1.3) is output controllable if and only if each row coefficient matrix corresponding to each row of the system transfer function W(s) is independent of each of the other row coefficient matrices. The theorems presented in this chapter will be referred to in the following chapters.

In Chapter III a detailed discussion of the selective output invariance problem will be presented. As a solution of this problem, a sufficient condition of the selective output invariance will be given.

Chapter IV will present the solution of the decoupling problem. The mathematical approach in terms of row coefficient matrices will reduce the effort required to derive the necessary and sufficient conditions for decoupling which were originally developed by Falb and Wolovich [7]. In order to determine how closely the system (1.3) can be designed to the decoupled form if the necessary and sufficient conditions are not satisfied, the concept of selective decoupling will be defined, and necessary and sufficient conditions for selective decoupling will be derived.

In Chapter V, the minimal input-output interaction problem will be considered. Even though a general solution to this problem will not be obtained, a couple of special cases of selecting the matrices G and F will be presented with an illustrative example.

Chapter VI will be devoted to a discussion of the contribution of the decoupling property of the system (1.3) to some other system characteristics, such as stabilizability, parameter insensitivity, and functional reproducibility.

Conclusions and suggestions for further research will be presented in Chapter VII.

CHAPTER II

MATHEMATICAL PRELIMINARIES

The concept of "row coefficient matrices" is introduced in order to provide a mathematical tool for the analysis and design of noninteracting systems. To show an application of the notion of the row coefficient matrix, the output controllability criterion of the system (1.3) is considered. In fact, the system (1.3) can be represented by a simple form in terms of row coefficient matrices. This new representation of the system (1.3) will prove to be useful in giving intuitive explanations for the solution procedure to noninteraction problems.

2.1 Row Coefficient Matrix

Consider a transfer function matrix $W(s) = [w_{ij}(s)]$ of the system (1.1). The elements of W(s) are rational functions of s with real coefficients of the form

$$w_{ij}(s) = x_{ij}(s)/q(s)$$

where q(s) is the system characteristic equation, a polynomial function in s of degree n, and where $x_{ij}(s)$ are real polynomial functions of degree at most of n-1. It is easy to verify that the set

$$P_{n-1}(R) = \{\frac{1}{q(s)}(a_o+a_1 s + \dots + a_{n-1}s^{n-1}) \mid a_o, a_1, \dots, a_{n-1} \epsilon R\}$$

is a vector space over R under the usual addition and scalar multiplication. The dimension of $P_{n-1}(R)$ is equal to n, where

$$[\frac{1}{q(s)}, \frac{s}{q(s)}, \dots, \frac{s^{n-1}}{q(s)}]$$

is a "natural" basis for $P_{n-1}(R)$. It can also be verified that the set V of all r-tuples of polynomials in the set $P_{n-1}(R)$, i. e.,

$$V = \{[p_{i1}(s), p_{i2}(s), \dots, p_{ir}(s)] \mid p_{ik}(s) \epsilon P_{n-1}(R), k = 1,2,\dots,r\}$$

is a vector space of dimension nr. Notice that the i-th row of W(s) is described by an element of V.

Denote the i-th row of W(s) by

$$w_i(s) = [\frac{x_{i1}(s)}{q(s)}, \frac{x_{i2}(s)}{q(s)}, \dots, \frac{x_{ir}(s)}{q(s)}] \tag{2.1}$$

$$= [\frac{1}{q(s)}(a^i_{10}+a^i_{11}s+a^i_{12}s^2+ \dots +a^i_{1(n-1)}s^{n-1}), \frac{1}{q(s)}(a^i_{20}+a^i_{21}s$$

$$+ a^i_{22}s^2+\dots+ a^i_{2(n-1)}s^{n-1}), \dots, \frac{1}{q(s)}(a^i_{r0} + a^i_{r1}s$$

$$+ a^i_{r2}s^2+ \dots + a^i_{r(n-1)}s^{n-1})].$$

where $a^i_{jk} \varepsilon R$ for $i = 1, 2, \ldots n$, $j = 1, 2, \ldots, r$, and $k = 0, 1, \ldots, n-1$.

Define a row vector S as

$$S = [\frac{1}{q(s)}, \frac{s}{q(s)}, \frac{s^2}{q(s)}, \ldots, \frac{s^{n-1}}{q(s)}].$$

Then $w_i(s)$ of equation (2.1) can be written in the form

$$w_i(s) = S\ E_i$$

where $(n \times r)$ matrix E_i will be referred to as the row coefficient matrix (RCM) of the i-th row of W(s), i. e.,

$$E_i = \begin{bmatrix} a^i_{10} & a^i_{20} & \cdots & a^i_{r0} \\ a^i_{11} & a^i_{21} & \cdots & a^i_{r1} \\ \vdots & \vdots & & \vdots \\ a^i_{1(n-1)} & a^i_{2(n-1)} & \cdots & a^i_{r(n-1)} \end{bmatrix}$$

Let the collection of all $(n \times r)$ matrices be Ω. Clearly Ω is a vector space of dimension nr. We know that two finite dimensional vector spaces over the same field, which have the same dimension, are isomorphic. Hence Ω and V are isomorphic.

Example: Suppose that the matrix W(s) is given by

$$W(s) = \frac{1}{q(s)} \begin{bmatrix} 1 + s + 5s^2 & 2s + 4s^2 & 1 + 3s + 9s^2 \\ 2 + 3s + 5s^2 & 2s + 4s^2 & 4 + 3s + 9s^2 \\ s & 2s & 3s \end{bmatrix} \qquad (2.2)$$

where $q(s) = 1 + 2s + 3s^2 + 4s^3$. Denote the RCM for the i-th row vector of W(s) by E_i. Then for the given matrix W(s) in equation (2.2), we have

$$E_1 = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 3 \\ 5 & 4 & 9 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 2 & 0 & 4 \\ 3 & 2 & 3 \\ 5 & 4 & 9 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}.$$

Defining the vector S by $S = [\frac{1}{q(s)}, \frac{s}{q(s)}, \frac{s^2}{q(s)}]$, W(s) can be written in the form

$$W(s) = \begin{bmatrix} S\,E_1 \\ S\,E_2 \\ S\,E_3 \end{bmatrix}.$$

Likewise, we can define a column coefficient matrix (CCM) by introducing an (n x 1) column vector S' (instead of row vector S). The system W(s) can also be described in terms of the CCM's.

2.2 System Representation in Terms of RCM's

Now we shall attempt to describe the system (1.3) in terms of the RCM's. By extending the well-known expansion for $(sI - A)^{-1}$

(cf. [9], p. 82-85) to $(sI - A - BF)^{-1}$, we have

$$w_i(s) = [q(s)]^{-1}(C_iR_os^{n-1}+C_iR_1s^{n-2}+ \ . \ . \ . \ + C_iR_{n-1})BG$$

where

$$q(s) = s^n - q_1s^{n-1} - q_2s^{n-2} - . \ . \ . \ -q_n = \det\ (sI - A - BF) \neq 0$$

$$R_o = (A + BF)^o = I_n \ ((n \times n) \text{ identity matrix})$$

$$R_j = (A + BF)^j - \sum_{k=1}^{j} a_k \ (A + BF)^{j-k} \quad \text{for } j = 1, 2, . \ . \ ., n-1$$

and

C_i is the i-th row of matrix C.

Letting

$$S = [\frac{1}{q(s)}, \frac{s}{q(s)}, \frac{s^2}{q(s)}, . \ . \ ., \frac{s^{n-1}}{q(s)}],$$

$w_i(s)$ can be represented in the form

$$w_i(s) = S \begin{bmatrix} C_i \ R_{n-1} \ BG \\ C_i \ R_{n-2} \ BG \\ \vdots \\ C_i \ R_1 BG \\ C_i \ R_o \ BG \end{bmatrix} \tag{2.3}$$

Therefore, the RCM E_i for the i-th row of the system W(s) is given by

$$E_i = \begin{bmatrix} C_i R_{n-1} BG \\ C_i R_{n-2} BG \\ \vdots \\ C_i R_1 BG \\ C_i R_o BG \end{bmatrix} . \tag{2.4}$$

The E_i in equation (2.4) can also be expressed in the following form:

$$E_i = \begin{bmatrix} -q_{n-1} & -q_{n-2} & \cdots & -q_1 & 1 \\ -q_{n-2} & -q_{n-3} & \cdots & 1 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ -q_1 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix} \begin{bmatrix} C_i B \\ C_i (A+BF)B \\ \vdots \\ C_i (A+BF)^{n-1}B \end{bmatrix} G \tag{2.5}$$

If Γ_i is defined by

$$\Gamma_i = \begin{bmatrix} C_i B \\ C_i (A+BF)B \\ \vdots \\ C_i (A+BF)^{n-1}B \end{bmatrix} \tag{2.6}$$

and Q is defined by

$$Q = \begin{bmatrix} -q_{n-1} & -q_{n-2} & \cdots & -q_1 & 1 \\ -q_{n-2} & -q_{n-3} & \cdots & 1 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ -q_1 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix} \tag{2.7}$$

the RCM E_i is described by

$$E_i = Q\Gamma_i G. \tag{2.8}$$

Then the i-th row of W(s) is finally represented by

$$w_i(s) = SE_i = SQ\Gamma_i G. \tag{2.9}$$

By repeating the preceding discussion for each row of W(s), the system transfer function matrix W(s) can be given in the form

$$W(s) = \begin{bmatrix} SQ\Gamma_1 \\ SQ\Gamma_2 \\ \vdots \\ SQ\Gamma_m \end{bmatrix} G \tag{2.10}$$

If the matrix G is denoted by column vectors, g_1, g_2, ..., g_r as

$$G = [g_1, g_2, \dots, g_r], \tag{2.11}$$

the system transfer function matrix in equation (2.10) becomes

$$W(s) = \begin{bmatrix} SQ\Gamma_1 \\ SQ\Gamma_2 \\ \vdots \\ SQ\Gamma_m \end{bmatrix} [g_1, g_2, \ldots, g_r]$$

$$= \begin{bmatrix} SQ\Gamma_1 g_1 & SQ\Gamma_1 g_2 & \cdots & SQ\Gamma_1 g_r \\ SQ\Gamma_2 g_1 & SQ\Gamma_2 g_2 & \cdots & SQ\Gamma_2 g_r \\ \vdots & \vdots & & \vdots \\ SQ\Gamma_m g_1 & SQ\Gamma_m g_2 & \cdots & SQ\Gamma_m g_r \end{bmatrix}. \tag{2.12}$$

Equation (2.12) is thus the representation of the system (1.3) in terms of the RCM's. In the next chapter, we will start the discussion of the selective output invariance problem by using the system representation given by equation (2.12).

2.3 Output Controllability Criterion in Terms of RCM's

This section is devoted to the discussion of an interesting result for the output controllability of the system (1.3), in which the RCM's play an important role. The necessary and sufficient condition of the output controllability to the system (1.3) can be expressed (see [36]) by

$$\text{rank}\,[CBG,\ C(A+BF)BG,\ \ldots,\ C(A+BF)^{n-1}BG] = m$$

which is equivalent to saying that the system (1.3) is output controllable if and only if

$$\sum_{i=1}^{m} k_i [C_i BG,\ C_i(A+BF)BG,\ \ldots,\ C_i(A+BF)^{n-1}BG] = \underline{0}$$

implying that

$$k_1 = k_2 = \ldots = k_m = 0 \tag{2.13}$$

where C_i is the i-th row of matrix C, the k_i are scalars and $\underline{0}$ is a (1 x nr) zero vector. We can further restate the criterion (2.13) as follows: The system (1.3) is output controllable if and only if

$$\begin{array}{lllll} k_1 C_1 B G & + k_2 C_2 B G & + \ldots + k_m C_m B G & = \underline{0} \\ k_1 C_1 (A+BF)BG & + k_2 C_2 (A+BF)BG & + \ldots + k_m C_m (A+BF)BG & = \underline{0} \\ \vdots & & & \vdots \\ k_1 C_1 (A+BF)^{n-1}BG & + k_2 C_2 (A+BF)^{n-1}BG & + \ldots + k_m C_m (A+BF)^{n-1}BG & = \underline{0} \end{array} \tag{2.14}$$

has only solution $k_1 = k_2 = \ldots = k_m = 0$, where $\underline{0}$ is (1 x r) zero vector. Since Γ_i was defined in equation (2.6) as

$$\Gamma_i = \begin{bmatrix} C_i B \\ C_i(A+BF)B \\ \vdots \\ C_i(A+BF)^{n-1}B \end{bmatrix},$$

the necessary and sufficient condition of output controllability is finally described by the statement that

$$\sum_{i=1}^{m} k_i \Gamma_i G = 0 \tag{2.15}$$

has only solution $k_1 = k_2 = \dots = k_m = 0$, where (0) is an (n x r) zero matrix.

In equation (2.9) of the previous section, we showed that the i-th row of the system transfer function matrix W(s) is described by

$$w_i(s) = SQ\Gamma_i G. \tag{2.16}$$

Construct a linear combination of the $w_i(s)$ and set it equal to zero, i. e.,

$$\sum_{i=1}^{m} k_i' \, w_i(s) = S(\sum_{i=1}^{m} k_i' Q\Gamma_i G) = \underline{0} \tag{2.17}$$

where k_i' are scalars and $\underline{0}$ is a (1 x r) zero vector.

Now we will show that the criterion (2.15) is equivalent to stating that equation (2.17) has only the solution $k_1' = k_2' = \dots = k_m' = 0$. Since in equation (2.17) $S \neq \underline{0}$, equation (2.17) is reduced to

$$\sum_{i=1}^{m} k_i' \; Q \; \Gamma_i G = (0) \tag{2.18}$$

where (0) is an (n x r) zero matrix. Since Q is nonsingular, it follows that

$$\sum_{i=1}^{m} k_i' \; \Gamma_i G = (0). \tag{2.19}$$

By comparing equation (2.19) with equation (2.15), we will notice that in equation (2.19) $k_1' = k_2' = \ldots = k_m' = 0$ turns out to be the necessary and sufficient condition for the system output controllability. Then the following theorems are established.

Theorem 2.1

The system (1.3) is output controllable if and only if the rows of W(s) are linearly independent.

From the isomorphism of Ω onto V, the following theorem is naturally derived.

Theorem 2.2

The system (1.3) is output controllable if and only if the RCM's of W(s), E_1, E_2, ..., E_m are linearly independent of each other.

Suppose that G is assumed to be an (r x r) nonsingular matrix. In this case, both equation (2.19) and equation(2.15) hold if and only if

$$\sum_{i=1}^{m} k_i' \Gamma_i = 0$$

and

$$\sum_{i=1}^{m} k_i \Gamma_i = 0,$$

respectively. The third theorem for the output controllability of the system (1.3) is stated as follows:

<u>Theorem 2.3</u>

When the matrix G is nonsingular, the system (1.3) is output controllable if and only if the matrices Γ_i $i = 1, 2, \ldots, m$ are linearly independent of each other.

CHAPTER III

SELECTIVE OUTPUT INVARIANCE PROBLEM

A discussion of the selective output invariance property was first introduced by Zunde [37] under the subject of "basic interactions" in 1968. His study was developed for the linear time-invariant system described by equation (1.1). The main idea of this study was to investigate the existence or nonexistence of interaction between sets of input, output and state variables. In this chapter, we will confine ourselves to the selective output invariance problem which can be explained as the investigation of noninteraction conditions between input and output variables of the system (1.3).

3.1 Previous Results

An interesting theorem related to the selective output invariance problem presented by Zunde [37] will be reviewed. Zunde's definition of selective output invariance was already described in Chapter I. It was also commended in Chapter I that the linear time-invariant system is selectively i-th output invariant with respect to the j-th input if and only if the (i,j)-th element of its system transfer function matrix takes a value of zero. Specifically, for the system (1.6), denote the matrix (sI-A) by Z(s) where s is a variable in Laplace transform. The determinant of the matrix Z(s) is given by

$$\det Z(s) = \det (sI-A).$$

Define

$$ {}^{i}Z^{j}(s)=\begin{bmatrix} s-a_{11} & \cdots & -a_{1(i-1)} & b_{1j} & -a_{1(i+1)} & \cdots & -a_{1n} \\ -a_{21} & \cdots & -a_{2(i-1)} & b_{2j} & -a_{2(i+1)} & \cdots & -a_{2n} \\ \vdots & & \vdots & \vdots & \vdots & & \vdots \\ -a_{n1} & \cdots & -a_{n(i-1)} & b_{nj} & -a_{n(i+1)} & \cdots & s-a_{nn} \end{bmatrix},$$

that is, ${}^{i}Z^{j}(s)$ is given by substituting the i-th column of Z(s) by the j-th column of the matrix B. One of the main results presented by Zunde will be stated as follows:

<u>Theorem 3.1</u>

The system (1.1) is selectively i-th output invariant with respect to the j-th input if and only if

$$c_{i1} \det {}^{1}Z^{j}(s) + c_{i2} \det {}^{2}Z^{j}(s) + \ . \ . \ . + c_{in} \det {}^{n}Z^{j}(s) = 0$$

where c_{ij} is the (i,j)-th element of the matrix C.

Although Zunde's result seems to provide us a good starting point from which to approach the selective output invariance problem, the calculation of determinants for large values of n is difficult. Notice that for the system (1.3), Z(s) will be defined by

$$Z(s) = (sI - A - BF) \tag{3.1}$$

and ${}^{i}Z^{j}(s)$ will be given by substituting the i-th column of Z(s) in equation (3.1) by the j-th column of matrix BG. Using this Z(s), we must find the appropriate matrices G and F which satisfy the condition of Theorem 3.1.

3.2 Selective Output Invariance Problem

In this section, the selective output invariance problem is solved with the aid of the RCM. In equation (2.12) of Chapter II, the system (1.3) was represented in terms of RCM's, i. e.,

$$W(s) = \begin{bmatrix} SQ\Gamma_1 g_1 & SQ\Gamma_1 g_2 & \cdots & SQ\Gamma_1 g_r \\ SQ\Gamma_2 g_1 & SQ\Gamma_2 g_2 & \cdots & SQ\Gamma_2 g_r \\ \vdots & \vdots & & \vdots \\ SQ\Gamma_m g_1 & SQ\Gamma_m g_2 & \cdots & SQ\Gamma_m g_r \end{bmatrix}.$$

Since we are interested in the selective i-th output invariance with respect to the j-th input, only the (i,j)-th element of W(s) is considered in the following discussion. From the definition of selective output invariance the system (1.3) is said to be selectively i-th output invariant with respect to the j-th input if and only if the (i,j)-th element of W(s) takes a value of zero, i. e.,

$$SQ\Gamma_i g_j = 0 \tag{3.2}$$

where 0 is a scalar zero. Clearly, the left hand side of equation (3.2) takes a value of zero if and only if

$$Q\Gamma_i g_j = \underline{0} \tag{3.3}$$

where $\underline{0}$ is an (n x 1) zero vector. Equation (3.3) implies that all coefficients of s^i i = 1, 2, . . ., n-1 (s^i are elements of the vector s) take a value of zero. From the nonsingularity of the matrix Q defined by equation (2.7), equation (3.3) holds if and only if

$$\Gamma_i g_j = \underline{0} \tag{3.4}$$

We consider the problem to select the elements of the vector g_j which satisfies equation (3.4), provided that the matrix Γ_i is known. A vector $g_j = \underline{0}$ would be a solution to our problem, but this trivial solution is not of interest. It is desirable to obtain a nonzero solution g_j satisfying equation (3.4). By applying the well known result of linear algebra (see [12]*), we conclude that the system (1.3) is selectively i-th output invariant with respect to the j-th input if and only if the (n x r) matrix Γ_i is not of rank r. This is summarized in the following theorem:

---

* Consider the system $A\underline{x} = \underline{0}$ where A is an (m x n) matrix (such that $m \geq n$) and $\underline{x}$ is an (n x 1) vector. Then: (1) When the matrix A has rank n, only a trivial solution exists, that is, $\underline{x} = \underline{0}$; and (2) when the matrix A has rank p (<n), there exists (n-p) independent solutions for $\underline{x}$.

Theorem 3.2

The system (1.3) is selectively i-th output invariant with respect to the j-th input if and only if Γ_i is not of rank r.

Now we consider the condition for the existence of the feedback compensator F. Define $d_1, d_2, \ldots, d_m$ by

$$d_i = \min \{j : C_i A^j B \neq 0 \quad \text{for } j = 0, 1, \ldots, n-1\}$$

or

$$d_i = n-1 \text{ if } \quad C_i A^j B = 0 \quad \text{for all } j \tag{3.5}$$

for i = 1, 2, ..., m. If we could find a matrix F such that the matrix Γ_i given by

$$\Gamma_i = \begin{bmatrix} C_i B \\ C_i (A+BF)^1 B \\ \vdots \\ C_i (A+BF)^{n-1} B \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ C_i (A+BF)^{d_i} B \\ C_i (A+BF)^{d_i+1} B \\ \vdots \\ C_i (A+BF)^{n-1} B \end{bmatrix} \tag{3.6}$$

$$
= \begin{bmatrix} 0 \\ \vdots \\ 0 \\ C_i A^{d_i} B \\ C_i A^{d_i}(A+BF)^1 B \\ \vdots \\ C_i A^{d_i}(A+BF)^{n-d_i-1} B \end{bmatrix} \left.\vphantom{\begin{matrix} C \\ C \\ \vdots \\ C \end{matrix}}\right\} (n-d_i)\text{rows}
$$

is not of rank r, the selective output invariance property holds according to Theorem 3.2. The second and third equalities of equation (3.6) are proven in Appendix A. The observation of equation (3.6) leads to the conclusion that if $n-d_i < r$ (or $d_i > n-r$), the system (1.3) is selectively i-th output invariant with respect to the j-th input no matter how one chooses the elements of the matrix F. This property was originally introduced by Gilbert [11] as an F-invariance and defined as follows:

Definition

An F-invariance of the system (1.3) is any property of the system which for any fixed G does not depend on F.

Next it is necessary to examine whether or not we can find an appropriate matrix F such that the matrix Γ_i is not of rank r. The remainder of this section will lead the conclusion that such a matrix F always exists if

(1) $m \leq r$, that is, the number of input components is larger

than or equal to the number of output components, and

(2) for a specified i, $C_iA^{d_i}B \neq 0$.

Assume that for a specified i, $C_iA^{d_i}B \neq 0$. Define a matrix D^{-1} by

$$D^{-1} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_{i-1} \\ C_iA^{d_i}B \\ \alpha_{i+1} \\ \vdots \\ \alpha_r \end{bmatrix} \tag{3.7}$$

where α_j $j = 1, 2, \ldots, i-1, i+1, \ldots, r$ are $(1 \times r)$ vectors chosen so that the matrix D^{-1} is nonsingular. Make sure that the term $C_iA^{d_i}B$ is assigned to the i-th row of D^{-1}. The matrix F can be selected as follows:

$$F = -D \cdot \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_{i-1} \\ C_iA^{d_i+1} \\ \beta_{i+1} \\ \vdots \\ \beta_r \end{bmatrix} \tag{3.8}$$

where β_j $j = 1, 2, \ldots, i-1, i+1, \ldots, r$ are arbitrarily chosen $(1 \times n)$ vectors. The β_j can be zero vectors. The term $C_i A^{d_i+1}$ must be assigned to the i-th row of the most right-hand side matrix of equation (3.8). Then we can assert that for the matrix F in equation (3.8),

$$\text{rank}\ \Gamma_i = 1. \tag{3.9}$$

This is sufficient to meet the condition that Γ_i is not of rank r. Equation (3.9) is proved as follows: Consider the matrix Γ_i given by equation (3.6).

$$\Gamma_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ C_i A^{d_i} B \\ C_i A^{d_i}(A+BF)B \\ \vdots \\ C_i A^{d_i}(A+BF)^{n-d_i-1}B \end{bmatrix}.$$

Evaluate a quantity $C_i A^{d_i}(A+BF)$ in Γ_i with the matrix F given by equation (3.8).

$$C_i A^{d_i}(A + BF) = C_i A^{d_i+1} + C_i A^{d_i+1} BF$$

$$= C_i A^{d_i+1} [D^{-1}]_i \cdot [-D] \cdot \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_{i-1} \\ C_i A^{d_i+1} \\ \beta_{i+1} \\ \vdots \\ \beta_r \end{bmatrix} = C_i A^{d_i+1} - C_i A^{d_i+1} = \underline{0}$$

where $[D^{-1}]_i$ is the i-th row vector of D^{-1}. Consequently, we obtain for $k = 1, 2, \ldots, n-d_i-1$,

$$C_i A^{d_i}(A+BF)^k B = C_i A^{d_i}(A+BF)(A+BF)^{k-1} B = \underline{0}.$$

Thus, Γ_i is written in the form of

$$\Gamma_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ C_i A^{d_i} B \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{3.10}$$

which obviously has rank one.

Finally, the selection of the matrix G will be considered. Define a matrix D* by

$$D^* = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_{i-1} \\ C_i A^{d_i} B \\ \delta_{i+1} \\ \vdots \\ \delta_r \end{bmatrix}, \tag{3.11}$$

where $C_i A^{d_i} B$ is assigned at any row other than the j-th row of D^*, and vectors δ_k $k = 1, \ldots, i-1, i+1, \ldots, r$ are arbitrarily chosen so that the matrix D^* becomes nonsingular. Then the matrix G is chosen as follows:

$$G = D^{*-1}. \tag{3.12}$$

It is now possible to state and prove a theorem for the selective output invariance problem.

<u>Theorem 3.3</u>

If $r \geq m$ and $C_i A^{d_i} B \neq 0$, there always exists the matrix G and the matrix F such that the system (1.3) is selectively i-th output invariant with respect to the j-th input. Such matrices G and F are given, for instance, by G in equation (3.12) and F in equation (3.8).

Proof:

For the matrix F in equation (3.8), Γ_i was given in equation (3.10) as

$$\Gamma_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ C_i A^{d_i} B \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

Then the i-th row of W(s) is given by

$$SQ\Gamma_i G = SQ\Gamma_i D^{*-1} = SQ \cdot \begin{bmatrix} 0 \\ \vdots \\ 0 \\ C_i A^{d_i} B \\ C \\ \vdots \\ 0 \end{bmatrix} \begin{bmatrix} \delta_1 \\ \vdots \\ \delta_{i-1} \\ C_i A^{d_i} B \\ \delta_{i+1} \\ \vdots \\ \delta_r \end{bmatrix}^{-1}$$

Since $C_i A^{d_i} B$ is not assigned at the j-th row of the matrix D^*, we obtain

$$SQ\Gamma_i G = SQ \begin{array}{c} \begin{bmatrix} & 0 & \\ & 0 & \\ * & \vdots & * \\ & 0 & \end{bmatrix} \\ \uparrow \\ \text{j-th column} \end{array} = \begin{array}{c} \begin{bmatrix} * & 0 & * \end{bmatrix} \\ \uparrow \\ \text{j-th element} \end{array} \qquad (3.13)$$

where asterisks denote elements except on the j-th column and the j-th element. We observe in equation (3.13) that the (i,j)-th element of W(s) takes a value of zero. Q.E.D.

As an example, we examine the conditions under which the second output component is invariant with respect to the third input component of the following system:

$$\dot{x}(t) = \begin{bmatrix} 3 & -1 & 1 \\ 2 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix} x(t) + \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 0 \\ 0 & 1 & 1 \end{bmatrix} x(t)$$

with the control law

$$u(t) = F\, x(t) + G\, v(t).$$

Since $C_2B = [8 \quad 5 \quad 3] \neq 0$, $d_2 = 0$. From equation (3.7), we obtain

$$D^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 8 & 5 & 3 \\ 0 & 1 & 0 \end{bmatrix}.$$

Then

$$D = \frac{1}{3}\begin{bmatrix} -3 & 0 & 0 \\ 0 & 0 & -3 \\ 8 & -1 & 5 \end{bmatrix}.$$

From equation (3.8), we have

$$F = -D\begin{bmatrix} 0 & 0 & 0 \\ & -C_2A & \\ 0 & 0 & 0 \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} -3 & 0 & 0 \\ 0 & 0 & -3 \\ 8 & -1 & 5 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 12 & -2 & 5 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\frac{12}{3} & \frac{2}{3} & -\frac{5}{3} \end{bmatrix}.$$

From equation (3.12), G is given by

$$G = \begin{bmatrix} 8 & 5 & 3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}^{-1} = \frac{1}{3}\begin{bmatrix} 0 & 3 & 0 \\ 0 & 0 & 3 \\ 1 & -8 & -5 \end{bmatrix}.$$

Therefore,

$$W(s) = C\ (sI-A-BF)^{-1}BG$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 0 \\ 0 & 1 & 1 \end{bmatrix} \left\{ sI - \begin{bmatrix} 3 & -1 & 1 \\ 2 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ -\frac{12}{3} & \frac{2}{3} & -\frac{5}{3} \\ -\frac{12}{3} & \frac{2}{3} & -\frac{5}{3} \end{bmatrix} \right\}^{-1} \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \frac{1}{3} \begin{bmatrix} 0 & 3 & 0 \\ 0 & 0 & 3 \\ 1 & -8 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 0 \\ 0 & 1 & 1 \end{bmatrix} \frac{1}{q(s)} \begin{bmatrix} s^2-s & -s & s \\ -2s+\frac{8}{3} & s^2-\frac{10}{3}s+4 & -\frac{2}{3}s \\ -3s+\frac{8}{3} & -\frac{1}{3}s+4 & s^2-\frac{11}{3} \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ \frac{1}{3} & -\frac{2}{3} & -\frac{2}{3} \\ \frac{1}{3} & -\frac{8}{3} & -\frac{5}{3} \end{bmatrix}$$

$$= \frac{1}{q(s)} \begin{bmatrix} \frac{1}{3}s^2 - \frac{4}{3}s + \frac{4}{3} & -\frac{5}{3}s^2 + 4s & \frac{2}{3}s^2 + \frac{4}{3}s \\ s^2 - 4s + 4 & 0 & 0 \\ \frac{2}{3}s^2 - \frac{8}{3}s + \frac{8}{3} & \frac{10}{3}s^2 + 9s & -\frac{7}{3}s^2 + \frac{42}{9}s \end{bmatrix} \tag{3.14}$$

where $q(s) = s^3 - 4s^2 + 4s = \det(sI-A-BF)$. Equation (3.14) shows that the given system is selectively second output invariant with respect to the third input.

In this chapter, the selective output invariance problem was investigated for the case that $r \geq m$. Since the decoupling property was described as an extension of the selective output invariance property in Chapter I, some of the results developed in this section will be referred to in the study of the decoupling problem in the next chapter.

Although the selective output invariance problem has been solved for the case of $r \geq m$, the problem for the case $r < m$ has not been considered. This topic is left as an area of future research work.

CHAPTER IV

DECOUPLING PROBLEM

In this chapter, the decoupling problem for the system (1.3) will be investigated through the application of the concept of the row coefficient matrix. The methodology used in this chapter for the derivation of a necessary and sufficient condition for the decoupling is basically different from Falb and Wolovich [7] and Wonham and Morse [35] because of the use of row coefficient matrices. The scope of the research effort devoted to the analysis of the decoupling problem is expanding to cover further generalizations of the problem such as partial decoupling [30] and exact model matching [34]. The results of our new approach will contribute to a better understanding of the overall decoupling problem and its newly emerging extensions. A new property called selective decoupling will be defined and investigated as a generalization of the decoupling problem.

4.1 Decoupling Problem

Consider the decoupling problem of the system (1.3). For the transfer function matrix of the system (1.3) described by equation (2.12), the number of output components is assumed to be exactly equal to the number of input components, that is, $m=r$. Then the matrix $W(s)$ is given by

$$W(s) = \begin{bmatrix} sQ\Gamma_1 g_1 & sQ\Gamma_1 g_2 & \cdots & sQ\Gamma_1 g_m \\ sQ\Gamma_2 g_1 & sQ\Gamma_2 g_2 & \cdots & sQ\Gamma_2 g_m \\ \vdots & \vdots & & \vdots \\ sQ\Gamma_m g_1 & sQ\Gamma_m g_2 & \cdots & sQ\Gamma_m g_m \end{bmatrix}.$$

From the definition of the decoupling property, for the system (1.3) to be decoupled the matrix $W(s)$ must have a diagonal form, and diagonal elements must be all nonzero rational functions of s. If the system $W(s)$ is diagonalized but takes one or more zero rationals of s on the diagonal, $W(s)$ will have zero row (s) and so the system will not be able to be decoupled. From equation (2.9), the existence of a zero row at the i-th row of $W(s)$ corresponds to the existence of a zero matrix Γ_i and/or the existence of a zero matrix G. Therefore, as a necessary condition for decoupling of the system (1.3), there must exist nonzero matrices Γ_i $i = 1, 2, . . . , m$ and a nonzero matrix G.

Paul [26] presents another necessary condition for decoupling as follows:

Theorem 4.1

The necessary conditions for decoupling of the system (1.3) are that (1) the matrix C must be of rank m; and (2) the matrix B must be of rank m (where $m = r$).

These requirements that rank $C = m$ and rank $B = m$ are generally fundamental assumptions for the system (1.3). If rank B is less than m, some inputs are redundant and can be discarded. Similar comments apply to the case where rank C is less than m.

Now we design the appropriate matrices F and G so that the transfer function matrix W(s) has a diagonal form. First consider the problem of selecting a matrix G. It was shown in equation (3.4) that the (i,j)-th element of W(s) takes a value of zero if and only if

$$\Gamma_i g_j = \underline{0}$$

where g_j is the j-th column vector of the matrix G. Thus, clearly the system is decoupled, if and only if for each i (i = 1, 2, . . ., m) and j (j = 1, 2, ..., m)

$$\Gamma_i g_j = \underline{0} \qquad \text{for } i \neq j, \tag{4.1}$$

$$\Gamma_i g_j \neq \underline{0} \qquad \text{for } i = j. \tag{4.2}$$

The matrix G to be selected consists of the nonzero vectors g_1, g_2, ..., g_m satisfying equation (4.1) and equation (4.2).

The following theorem is immediately derived from equation (4.1) and equation (4.2).

<u>Theorem 4.2</u>

The system (1.3) can be decoupled if and only if rank E = m and rank $E_{K(\{i\})} = m-1$ for $i = 1, 2, \ldots, m$ where E and $E_{K(\{i\})}$ are defined by the following augmented matrices:

$$E = \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \vdots \\ \Gamma_m \end{bmatrix} \tag{4.3}$$

$$E_{K(\{i\})} = \begin{bmatrix} \Gamma_1 \\ \vdots \\ \Gamma_{i-1} \\ \Gamma_{i+1} \\ \vdots \\ \Gamma_m \end{bmatrix} \tag{4.4}$$

The vector g_i, the i-th column vector of the matrix G, is a solution of the equation $E_{K(\{i\})}\, g_i = \underline{0}$.

Proof:

Assume that $W(s)$ is decoupled, that is, that equation (4.1) and equation (4.2) hold. First we show that for the system (1.3) to be decoupled, the matrix G must be nonsingular. Assume that the j-th column vector of G, g_j, to be described by a linear combination of $g_1, g_2, \ldots, g_{j-1}, g_{j+1}, \ldots, g_m$, that is,

$$g_j = c_1g_1 + c_2g_2 + \ldots + c_{j-1}g_{j-1} + c_{j+1}g_{j+1} + \ldots + c_mg_m \qquad (4.5)$$

Multiplying both sides of equation (4.5) by the matrix Γ_j from the left, we obtain

$$\Gamma_jg_j = \Gamma_jc_1g_1 + \Gamma_jc_2g_2 + \ldots + \Gamma_jc_{j-1}g_{j-1} + \Gamma_jc_{j+1}g_{j+1} + \ldots + \Gamma_jc_mg_m \qquad (4.6)$$

Since every term on the right hand side of equation (4.5) becomes zero by equation (4.1), equation (4.6) is reduced to

$$\Gamma_jg_j = \underline{0}$$

which contradicts the decoupling condition (4.2). Therefore, vectors $g_1, g_2, \ldots, g_m$ must be linearly independent for decoupling.

To show rank E = m, consider the linear equation

$$E\ g = \underline{0} \qquad (4.7)$$

where E is defined in equation (4.3) and g is an (m x 1) vector described by a linear combination of $g_1, g_2, \ldots, g_m$. Then equation (4.7) is written in the form

$$E\ (c_1g_1 + c_2g_2 + \ldots + c_mg_m) = \underline{0}. \qquad (4.8)$$

Equation (4.8) is restated as

$$
\begin{aligned}
\Gamma_1 (c_1 g_1 + c_2 g_2 + \cdot \cdot \cdot + c_m g_m) &= \underline{0} \\
\Gamma_2 (c_1 g_1 + c_2 g_2 + \cdot \cdot \cdot + c_m g_m) &= \underline{0} \\
&\vdots \\
\Gamma_m (c_1 g_1 + c_2 g_2 + \cdot \cdot \cdot + c_m g_m) &= \underline{0}.
\end{aligned}
\tag{4.9}
$$

Substitution of equation (4.1) and equation (4.2) into equation (4.9) yields

$$c_1 = c_2 = \cdot \cdot \cdot = c_m = 0.$$

This implies that in equation (4.7) only zero vector g can be a solution of the linear equation (4.7). Then

$$\text{rank } E = m.$$

To show rank $E_{K(\{i\})} = m-1$, consider the linear equation

$$E_{K(\{i\})}\, g = \underline{0} \tag{4.10}$$

where $E_{K(\{i\})}$ is defined in equation (4.2) and g is an (m x 1) vector described by a linear combination of $g_1, g_2, \ldots, g_m$. Then equation (4.10) is written in the form

$$E_{K(\{i\})}\ (c_1g_1 + c_2g_2 + \dots + c_mg_m) = \underline{0}$$

which is also written as

$$\begin{aligned} \Gamma_i\ (c_1g_1 + c_2g_2 + \dots + c_mg_m) &= \underline{0} \\ &\vdots \\ \Gamma_{i-1}\ (c_1g_1 + c_2g_2 + \dots + c_mg_m) &= \underline{0} \\ \Gamma_{i+1}\ (c_1g_1 + c_2g_2 + \dots + c_mg_m) &= \underline{0} \\ &\vdots \\ \Gamma_m\ (c_1g_1 + c_2g_2 + \dots + c_mg_m) &= \underline{0}. \end{aligned} \tag{4.11}$$

By substituting equation (4.1) and equation (4.2) into equation (4.11), we obtain

$$c_1 = c_2 = \dots = c_{i-1} = c_{i+1} \dots = c_m = 0.$$

This implies that all solutions for g satisfying equation (4.10) are given by the vector g_i multiplied by some scalar constant. Then

$$\text{rank } E_{K(\{i\})} = m-1$$

Similarly

$$\text{rank } E_{K(\{i\})} = m-1 \qquad \text{for each } i.$$

The sufficiency of Theorem 4.1 follows easily.

Q.E.D.

The following theorem shows a further simplification of Theorem 4.2. For a matrix A, spA represents a space spanned by all row vectors of the matrix A. The symbol $\Sigma\oplus$ implies the direct sum of the spaces listed after the symbol. Denote the dimension of the spA by dim(spA).

Theorem 4.3

The system (1.3) can be decoupled if and only if

$$\text{rank } \Gamma_i = 1 \qquad \text{for } i = 1, 2, \ldots, m$$

and

$$\text{rank} \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_m \end{bmatrix} = m$$

where γ_i is a $(1 \times m)$ row vector selected from the nonzero rows of Γ_i.

A lemma is presented next because the proof of Theorem 4.3 evolves directly from the following lemma.

Lemma 4.1

The system (1.3) can be decoupled, that is, rank E = m and rank $E_{K(\{i\})} = m-1$ for i = 1, 2, ..., m, if and only if rank $\Gamma_i = 1$ for i = 1, 2, ..., m and $spE = \sum_{i=1}^{m}\oplus\, sp\Gamma_i$.

Proof:

The necessity of lemma 4.1 is proved as follows: Assume that $sp\Gamma_i \neq \{0\}$ for each i. Now, rank $E_{K(\{i\})} = m-1$ for any i and in particular for i = 1 rank $E_{K(\{1\})} = m-1$. Then

$$\dim (spE_{K(\{1\})}) = \dim (sp\Gamma_2 + sp\Gamma_3 + \ldots + sp\Gamma_m) = m-1. \qquad (4.12)$$

We show that for $2 \leq j \leq m$

$$sp\Gamma_2 + sp\Gamma_3 + \ldots + sp\Gamma_j \subsetneqq sp\Gamma_2 + sp\Gamma_3 + \ldots + sp\Gamma_j + sp\Gamma_{j+1}. \qquad (4.13)$$

In equation (4.13), clearly the relationship "$\subset$" holds. Thus we prove that

$$sp\Gamma_2 + sp\Gamma_3 + \ldots + sp\Gamma_j \neq sp\Gamma_2 + sp\Gamma_3 + \ldots + sp\Gamma_j + sp\Gamma_{j+1}. \qquad (4.14)$$

Suppose that

$$sp\Gamma_2 + sp\Gamma_3 + \ldots + sp\Gamma_j = sp\Gamma_2 + sp\Gamma_3 + \ldots + sp\Gamma_j + sp\Gamma_{j+1}$$

then

$$sp\Gamma_{j+1} \subseteq sp\Gamma_2 + sp\Gamma_3 + \ldots + sp\Gamma_j. \tag{4.15}$$

Consider $spE_{K(\{j+1\})}$,

$$spE_{K(\{j+1\})} = sp\Gamma_1 + (sp\Gamma_2 + \ldots + sp\Gamma_j) + sp\Gamma_{j+2} + \ldots + sp\Gamma_m. \tag{4.16}$$

By equation (4.15), equation (4.16) is restated in the form of

$$spE_{K(\{j+1\})} = sp\Gamma_1 + (sp\Gamma_2 + \ldots + sp\Gamma_j + sp\Gamma_{j+1}) + sp\Gamma_{j+2} + \ldots + sp\Gamma_m$$

$$= spE$$

which contradicts the conditions of Lemma 4.1, that is, $spE = m \neq m-1 = spE_{K(\{j+1\})}$. Therefore equation (4.13) as well as equation (4.14) hold. From equation (4.13), we obtain

$$sp\Gamma_2 \subsetneqq (sp\Gamma_2 + sp\Gamma_3) \subsetneqq (sp\Gamma_2 + sp\Gamma_3 + sp\Gamma_4) \subsetneqq \cdots$$

$$\subsetneqq (sp\Gamma_2 + sp\Gamma_3 + \ldots + sp\Gamma_m) \tag{4.17}$$

which immediately leads us to

$$\dim (sp\Gamma_2) < \dim (sp\Gamma_2 + sp\Gamma_3) < \dim (sp\Gamma_2 + sp\Gamma_3 + sp\Gamma_4) < \cdots$$

$$< \dim (sp\Gamma_2 + sp\Gamma_3 + \ldots + sp\Gamma_m). \tag{4.18}$$

Since $sp\Gamma_i \neq \{0\}$ for each i, we assume that

$$\dim (sp\Gamma_2), \dim (sp\Gamma_3), \ldots, \dim (sp\Gamma_m) \geq 1.$$

If $\dim (sp\Gamma_2) > 1$, then from equation (4.18) we obtain

$$\dim (sp\Gamma_2 + sp\Gamma_3 + \ldots + sp\Gamma_m) \geq m$$

which contradicts equation (4.12). Therefore $\dim (sp\Gamma_2) = 1$. Thus rank $\Gamma_2 = 1$. A similar discussion can be repeated to prove rank $\Gamma_i = 1$ for $i = 1, 2, \ldots, m$. The second property of Lemma 4.1, that is, the equation

$$spE = \sum_{i=1}^{m} \oplus \, sp\Gamma_i$$

is proved from the fact that rank $E = m$ and rank $\Gamma_i = 1$ for each i. The sufficiency of Lemma 4.1 is self-evident.

Q.E.D.

The proof of Theorem 4.2 follow immediately from Lemma 4.1.

4.2 Interpretation of Falb and Wolovich's Result

Falb and Wolovich [7] presented a necessary and sufficient condition for decoupling the system (1.3) in a different way. The relationship between Theorem 4.3 and Falb and Wolovich's result is considered in this section.

Let G^* be an $(m \times m)$ matrix described by

$$G^* = \begin{bmatrix} C_1 A^{d_1} B \\ C_2 A^{d_2} B \\ \vdots \\ C_m A^{d_m} B \end{bmatrix} \tag{4.19}$$

where d_i, $i=1, 2, \ldots, m$ were defined by equation (3.5). Assume that the matrix G^* is nonsingular. Define G by

$$G = G^{*-1}. \tag{4.20}$$

Let F be

$$F = -G \begin{bmatrix} C_1 A^{d_1+1} \\ C_2 A^{d_2+1} \\ \vdots \\ C_m A^{d_m+1} \end{bmatrix} \tag{4.21}$$

Now, we shall show that the matrix G defined by equation (4.20) and the matrix F defined by equation (4.21) decouple the system (1.3). First, it will be shown that by substitution of the matrix F of equation (4.21) into the matrix Γ_i of equation (2.6) we obtain

$$\text{rank } \Gamma_i = 1 \quad \text{for } i = 1, 2, \ldots, m \tag{4.22}$$

which is the first condition of Theorem 4.3. The proof of equation (4.22) is briefly given as follows: Consider row vectors of the matrix Γ_i. By Lemma A.1 in Appendix A,

$$C_i(A + BF)^j B = C_i A^j B = \underline{0} \text{ for } j = 0, 1, 2, \ldots, d_i-1, \quad (4.23)$$

where $\underline{0}$ is a $(1 \times m)$ zero vector. For the term $C_i(A + BF)^{d_i+1} B$,

$$\begin{aligned} C_i(A + BF)^{d_i+1} &= C_i(A+BF)^{d_i}(A+BF) \\ &= C_i A^{d_i}(A+BF) = C_i A^{d_i+1} + C_i A^{d_i} BF \\ &= C_i A^{d_i+1} + [G^*]_i F \\ &= C_i A^{d_i+1} + [G^*]_i \cdot (-G) \cdot \begin{bmatrix} C_1 A^{d_1+1} \\ C_2 A^{d_2+1} \\ \vdots \\ C_m A^{d_m+1} \end{bmatrix} \\ &= C_i A^{d_i+1} - C_i A^{d_i+1} = \underline{0} \end{aligned}$$

where $\underline{0}$ is a $(1 \times n)$ zero vector and $[G^*]_i$ is the i-th row of the matrix G^*. Hence

$$C_i(A + BF)^{d_i+k} = \underline{0} \quad \text{for } k = 1, 2, \ldots, n-d_i-1 \quad (4.24)$$

From equations (4.23) and (4.24), we obtain

$$\Gamma_i = \begin{bmatrix} C_iB \\ C_i(A+BF)B \\ \vdots \\ C_i(A+BF)^{d_i}B \\ \vdots \\ C_i(A+BF)^{n-1}B \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ C_i(A+BF)^{d_i}B \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ C_iA^{d_i}B \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

where $C_iA^{d_i}B \neq 0$ because of the nonsingularity assumption of G*. Then Γ_i has rank one. The discussion is similarly repeated for all i. This completes the proof of equation (4.22).

The second condition of Theorem 4.3 is written simply by

$$\text{rank} \begin{bmatrix} C_1 A^{d_1} B \\ C_2 A^{d_2} B \\ \vdots \\ C_m A^{d_m} B \end{bmatrix} = m,$$

because γ_i defined in Theorem 4.3 is uniquely described by $C_iA^{d_i}B$ for each i.

Since two conditions of Theorem 4.3 are satisfied, the system (1.3) can be decoupled. The above results are summarized in the following theorem:

Theorem 4.4

The system (1.3) is decoupled by choosing G given by equation (4.20) and F given by equation (4.21), if and only if

$$\operatorname{rank}\begin{bmatrix} C_1 A^{d_1} B \\ C_2 A^{d_2} B \\ \vdots \\ C_m A^{d_m} B \end{bmatrix} = m.$$

One may notice that Theorem 4.4 is essentially the same as Theorem 1.1 of Falb and Wolovich. Finally we must show that the matrix G defined by equation (4.20) satisfies the condition

$$E_{K(\{i\})} g_i = \underline{0} \qquad \text{For } i = 1, 2, \ldots, m \tag{4.25}$$

which was stated in Theorem 4.1. Let $[G]^i$ represent the i-th column of the matrix G given by equation (4.20). Then the condition (4.25) is examined as follows: For $i = 1, 2, \ldots, m$,

$$E_{K(\{i\})} g_i = \begin{bmatrix} \Gamma_1 \\ \vdots \\ \Gamma_{i-1} \\ \Gamma_{i+1} \\ \vdots \\ \Gamma_m \end{bmatrix} [G]^i$$

$$
= \begin{bmatrix} C_1A^{d_1}B \\ 0 \\ \vdots \\ 0 \\ C_{i-1}A^{d_{i-1}}B \\ 0 \\ \vdots \\ 0 \\ C_{i+1}A^{d_{i+1}}B \\ 0 \\ \vdots \\ 0 \\ C_mA^{d_m}B \end{bmatrix} \left[\begin{pmatrix} C_1A^{d_1}B \\ C_2A^{d_2}B \\ \vdots \\ C_mA^{d_m}B \end{pmatrix}^{-1} \right]^{i} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}
$$

Then the matrix G given by equation (4.20) satisfies equation (4.25).

4.3 Selective Decoupling

According to Theorem 4.4, the system (1.3) can be decoupled if and only if

$$
\operatorname{rank} \begin{bmatrix} C_1A^{d_1}B \\ C_2A^{d_2}B \\ \vdots \\ C_mA^{d_m}B \end{bmatrix} = m \,. \tag{4.26}
$$

The primary purpose of this section is to determine what happens if the matrix in equation (4.26) has a rank less than m, and what conclusion can be drawn from this fact with regard to decoupling.

In order to generalize the problem, consider the system (1.3) with r inputs and m outputs. Throughout the discussion, we place the nonsingularity assumption on the matrix G. Assume that $r \geq m$.

Roughly speaking, if k output components (out of m) are independently controlled by k different input components, the system is said to be k-selectively decoupled. More precisely, the k-selective decoupling is defined in terms of W(s) of equation (2.12).

Definition

The system (1.3) is said to be selectively k decoupled for k input-output pairs if and only if for $i = 1, 2, \cdots, k$ and $j = 1, 2, \ldots, r$,

$$SQ\Gamma_i g_j \neq 0 \qquad \text{for } i = j,$$

$$SQ\Gamma_i g_j = 0 \qquad \text{for } i \neq j.$$

The definition of k-selective decoupling implies that the transfer function matrix W(s) of the system (1.3) is written in the form

$$W(s) = \begin{bmatrix} W_{11}(s) & W_{12}(s) \\ & \\ W_{21}(s) & W_{22}(s) \end{bmatrix}$$

where $W_{11}(s)$ is the $(k \times k)$ matrix with nonzero diagonal elements and

and zero off-diagonal elements, $W_{12}(s)$ is the $(k \times (r-k))$ zero matrix, and $W_{21}(s)$ and $W_{22}(s)$ are $((m-r) \times k)$, and $((m-r) \times (r-k))$ matrices whose elements are not specified.

By Theorem 3.3, if $r \geq m$ and $C_i A^{d_i} B \neq 0$ then there exist appropriate matrices G and F such that the system (1.3) is selectively i-th output invariant with respect to the j-th input. In fact, Theorem 3.3 is a sufficient condition for the selective output invariance problem. The proof of Theorem 3.3 actually shows that if $r \geq m$ and $C_i A^{d_i} B \neq 0$, the i-th row of $W(s)$ includes all zero elements except for one component whose position depends on the location of the row vector $C_i A^{d_i} B$ in the matrix D^* of equation (3.11).*

Let $\bar{K} = (1, 2, \ldots, k)$, $I \subset \bar{K}$, and $\bar{K}(I) = \bar{K} - I = \{x \in \bar{K} \mid x \notin I\}$. Assume that $k \geq 2$. The necessary and sufficient conditions for the k-selective decoupling of the system (1.3) are given in the following two theorems.

Theorem 4.5

The system (1.3) can be selectively k decoupled if and only if

$$\operatorname{rank} \bar{E} = k$$

and

---

* Under the nonsingularity assumption on G, the conditions $r \geq m$ and $C_1 A^{d_1} B \neq 0$ become necessary and sufficient for the system (1.3) to be selectively one decoupled.

$$\operatorname{rank} \bar{E}_{\bar{K}(\{i\})} = k-1 \quad \text{for } i = 1, 2, \ldots, k$$

where $\bar{E}$ and $\bar{E}_{\bar{K}(\{i\})}$ are defined by the following augmented matrices.

$$\bar{E} = \bar{E}_{K(\{i\})} = \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \vdots \\ \Gamma_k \end{bmatrix} \qquad \bar{E}_{\bar{K}(\{i\})} = \begin{bmatrix} \Gamma_1 \\ \vdots \\ \Gamma_{i-1} \\ \Gamma_{i+1} \\ \vdots \\ \Gamma_k \end{bmatrix}$$

The vector g_i, the i-th column vector of the matrix G, is a solution of the equation $\bar{E}_{\bar{K}(\{i\})} g_i = 0$.

Proof: The proof is similar to that of Theorem 4.2 and will not be presented here.

Theorem 4.6

The system (1.3) can be selectively k decoupled if and only if

$$\operatorname{rank} \Gamma_i = 1 \quad \text{for } i = 1, 2, \ldots, k$$

and

$$\operatorname{rank} \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_k \end{bmatrix} = k$$

where γ_i is a $(1 \times m)$ row vector selected from the nonzero rows of Γ_i.

Proof:

The proof is similar to Theorem 4.3 and Lemma 4.1.

The selection of the appropriate matrices G and F will be considered in a way similar to the one described for the decoupling problem in the previous section. Assume that $C_1A^{d_1}B$, $C_2A^{d_2}B$, ..., $C_kA^{d_k}B$ are nonzero vectors independent of each other. Let G^* be given by

$$G^* = \begin{bmatrix} C_1A^{d_1}B \\ \vdots \\ C_kA^{d_k}B \\ \alpha_{k+1} \\ \vdots \\ \alpha_r \end{bmatrix}$$

where α_j $j = k+1, \ldots, r$ are $(1 \times r)$ vectors being chosen in such a way that the matrix G^* is nonsingular. The matrix G is defined by

$$G = G^{*-1}. \qquad (4.27)$$

Let F be

$$F = -G \begin{bmatrix} C_1 A^{d_1+1} \\ C_2 A^{d_2+1} \\ \vdots \\ C_k A^{d_k+1} \\ \beta_k \\ \vdots \\ \beta_r \end{bmatrix} \tag{4.28}$$

where β_j, $j=k+1, \ldots, r$ are any $(1 \times n)$ row vectors. They can be zero vectors. By substituting equation (4.28) into the matrix F of Γ_i described by equation (2.6), we obtain

$$\Gamma_i = \begin{bmatrix} C_i B \\ C_i (A+BF) B \\ \vdots \\ C_i (A+BF)^{d_i} B \\ \vdots \\ C_i (A+BF)^{n-1} B \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ C_i A^{d_i} B \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{4.29}$$

for i = 1, 2, ..., k. The proof of the last equality of equation (4.29) is similar to that used in the development of the decoupling problem. Then Γ_i has a rank of one for i = 1, 2, ..., k, which completes the first condition of Theorem 4.6. The second condition of Theorem 4.6 is written simply by

$$\operatorname{rank}\begin{bmatrix} C_1A^{d_1}B \\ C_2A^{d_2}B \\ \vdots \\ C_kA^{d_k}B \end{bmatrix} = k,$$

because γ_i in Theorem 4.6 is uniquely described by $C_iA^{d_i}B$ for each i.

The above results are summarized as follows:

Theorem 4.7

Let $r \geq m$. The system (1.3) can be selectively k decoupled by choosing G given by equation (4.27) and F given by equation (4.28), if and only if

$$\operatorname{rank}\begin{bmatrix} C_1A^{d_1}B \\ C_2A^{d_2}B \\ \vdots \\ C_kA^{d_k}B \end{bmatrix} = k.$$

We can show that the matrix G defined by equation (4.27) satisfies (the condition)

$$\bar{E}_{\bar{K}(\{i\})}g_i = \underline{0} \qquad \text{for } i = 1, 2, \ldots, k \tag{4.30}$$

which was stated in Theorem 4.5. The verification of equation

(4.30) is similar to the one performed for the decoupling problem in the previous section.

CHAPTER V

MINIMAL INPUT-OUTPUT INTERACTION PROBLEM

5.1 Formulation fo the Minimal Input-Output Interaction Problem

Consider the minimal input-output interaction problem in terms of a geometric framework whose basic concept was originally introduced by Wonham and Morse [35]. The purpose of adopting this mathematical tool is to clarify the nature of noninteraction problems and to show the mathematical statements for the minimal input-output interaction problem.

From the definition of the minimal input-output interaction property as given in Chapter I, the compensated system (1.3) must be output controllable. The condition of the output controllability of the system (1.3) can be expressed in concrete form by [36]

$$\text{rank } P = m \tag{5.1}$$

where

$$P = [CBG,\ C(A+BF)BG,\ C(A+BF)^2BG,\ .\ .\ .\ ,\ C(A+BF)^{n-1}BG].$$

The study of the minimal input-output interaction problem must be based on concepts of selective output controllability and selective output invariance of the system. The selective output invariance of the system (1.3) was defined in Chapter I and the selective output invariance problem was completely solved in Chapter III. Here we recognize the fact that the selective output invariance and selective output controllability are quite closely related, as seen in the following definition of the selective output controllability of the system (1.3) (the definition of selective output invariance can be found on page 4).

Definition of Selective Output Controllability

Let T be a set of time at which the behavior of the system is defined, and let t_0, $t_1 \varepsilon T$ with $t_0 < t_1$. The i-th component of $y_i(t)$ of the output vector $y(t)$ is selectively output controllable with respect to the j-th component v_j of the input vector $v(t)$ at time t_0, if there exists some finite $t_1 > t_0$ and some input $v_j(t)$ which transfers any initial output $y_i(t_0)$ to any desired output $y_i(t_1)$, with $v_k(t) = 0$ for $k \neq j$, $k = 1, 2, \ldots, r$, during the time interval $[t_0, t_1]$.

For the linear time-invariant system (1.3), there exists the relationship such that the system is selectively output invariant if the system is not selectively output controllable, and vice versa [37]. The criterion for selective output controllability is given by Zunde and is stated as follows [38]:

Theorem 5.1

The system (1.3) is selectively i-th output controllable by the j-th input component if and only if there is at least one nonzero element in the following matrix P_{ij}:

$$P_{ij} = [C^{(i)}Bg_j,\ C^{(i)}(A+BF)Bg_j,\ C^{(i)}(A+BF)^2Bg_j, \ldots,\ C^{(i)}(A+BF)^{n-1}Bg_j]$$

where $C^{(i)}$ is the matrix C in which all rows except the i-th row are substituted by zero row vectors, and g_j is the j-th column vector of the matrix G.

Let $\tilde{E}^m$ be a real m-space. Denote the range of P by R(P) and the range of P_{ij} by $R(P_{ij})$. Zunde proved [37,38] that if the system is output controllable, any output vector in R(P) can be reached from the origin in a finite time by some control v. Likewise the $R(P_{ij})$ is interpreted as a reachable space from the origin by some j-th input v_j. The $R(P_{ij})$ is obviously a subspace of $\tilde{E}^m$. Because of the structure of P_{ij}, we can see that $R(P_{ij})$ must be either one-dimensional subspace of $\tilde{E}^m$ or zero subspace {0}. If $R(P_{ij})$ is one-dimensional subspace of $\tilde{E}^m$, the j-th input v_j can control the i-th output y_i. However, if $R(P_{ij})$ is zero subspace, no v_j can control y_i. Then Theorem 5.1 can be restated as follows:

Theorem 5.2

The system (1.3) is selectively i-th output controllable by the j-th input, if and only if $R(P_{ij}) = \tilde{E}^1$.

Since the system (1.3) is linear time-invariant, a similar theorem concerning the selective output invariance property will be established.

Theorem 5.3

The system (1.3) is selectively i-th output invariant by the j-th input, if and only if $R(P_{ij}) = \{\underline{0}\}$.

An immediate application of Theorem 5.2 and Theorem 5.3 will be the formulation of the decoupling problem. The problem is described in terms of P_{ij} as follows:

Decoupling Problem

For the decoupling problem, we choose a matrix G and a matrix F such that for the system (1.3)

$$R(P_{ij}) = \tilde{E}^1 \qquad \text{for } i = j$$

$$R(P_{ij}) = \{\underline{0}\} \qquad \text{for } i \neq j$$

where i = 1, 2, . . ., m and j = 1, 2, . . . , m.

The minimal input-output interaction problem will also be formulated in a similar manner. For this problem, the maximum number of selective output invariant elements must be generated in the system transfer function matrix W(s) by choosing appropriate G and F. In addition to this, the system must be output controllable. The latter condition is given by rank P = m of equation (5.1). Thus, the minimal input-output interaction problem is completely stated as follows:

Minimal Input-Output Interaction Problem

For the minimal input-output interaction problem, we choose a matrix G and a matrix F such that for the system (1.3) (1) $R(P) = E^m$; and (2) the maximum number of ordered pairs (i,j) satisfying $R(P_{ij}) = \{0\}$ is created for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, r$.

In the following two sections, the minimal input-output interaction problem in which the main concern is the selection of the appropriate matrices G and F is considered. A nonsingularity assumption will be placed on the matrix G for the discussion of section 5.3, but this assumption will be discarded for the discussion of section 5.2.

5.2 Selection of the Matrices G and F Without Nonsingularity Assumption on G

First of all, we should recognize that the system (1.3) which is minimally input-output interacting can include at most $m \times (r-1)$ zero elements. In other words, at least one nonzero element must exist in each row of $W(s)$ in order to retain the output controllability property of the system. An example of such systems including $m \times (r-1)$ zero elements is the decoupled system, assuming that $W(s)$ is a square matrix. Notice that for the decoupled system $W(s)$, if any one of the diagonal elements is substituted by zero, the output controllability condition does not hold. Then the decoupled system is minimally input-output interacting. However, the minimally input-output interacting system is not necessarily decoupled because for the minimal input-output interaction problem the number of input components may not be equal to the number of output components.

Consider a simple selection of the elements of the matrix G and the elements of the matrix F. Choose zero vectors $g_2, g_3, \ldots, g_r$, an appropriate vector g_1, and an appropriate matrix F so that vectors $\Gamma_i g_i$ are linearly independent for i = 1, 2,..., m. Then row coefficient matrices $Q\Gamma_i G$ i = 1, 2, ..., m of the system (1.3) become linearly independent (because the matrix Q is nonsingular). According to Theorem 2.2, the system (1.3) is output controllable. Furthermore, the matrix W(s) compensated by matrices G and F includes all nonzero elements in the first column and zero elements in the rest of the columns. Totally, m x (r-1) zero elements are generated in W(s). The matrices G and F selected in this way provide one solution to the minimal input-output interaction problem.

As a special case, suppose that the compensated system (1.1) is output controllable and a zero matrix is assigned to the matrix F. We can show that the minimal input-output interaction property of the system (1.3) is achieved only by the appropriate selection of elements of the matrix G. Before discussing the selection of G, it is shown that by the assignment of the zero matrix to F, Γ_i i = 1, 2, ..., m become linearly independent of each other.

Since the system (1.1) is output controllable, then according to Theorem 2.2, RCM's of the system (1.1)

$$Q \begin{bmatrix} C_i B \\ C_i AB \\ \vdots \\ C_i A^{n-1} B \end{bmatrix} \quad \text{for } i = 1, 2, \ldots, m$$

are linearly independent of each other, where the matrix Q is given by equation (2.7) with F being substituted by a zero matrix. Since the matrix Q is nonsingular, matrices

$$\begin{bmatrix} C_i B \\ C_i A B \\ \vdots \\ C_i A^{n-1} B \end{bmatrix} \quad \text{for } i = 1, 2, \ldots, m \quad (5.2)$$

are linearly independent of each other. Notice that the matrix in equation (5.2) is identical with the matrix Γ_i of equation (2.6) in which F is substituted by a zero matrix. Therefore, $\Gamma_i, i = 1, 2, \ldots, m$, are linearly independent of each other.

Under the assumption we have a zero matrix F, we consider the selection of the elements of the vector g_1. Using the result of Appendix B, if $\sum_{i=1}^{m} \Gamma_i$ has rank r, any vector g_1 whose components are all nonzero generates linearly independent vectors $\Gamma_1 g_1$, $\Gamma_2 g_2$, . . . , $\Gamma_m g_1$. Assign zero vectors to g_2, g_3, . . . , g_r. This choice of matrices G and F creates the output controllable matrix W(s), and exhibits all nonzero elements in the first column of W(s) and zero elements in the rest of the columns of W(s). Therefore, the system (1.3) is minimally input-output interacting. An illustration of this special case is given in the following example.

Example: Consider the system given by

$$W(s) = \frac{1}{q(s)} \begin{bmatrix} 1 + s + 5s^2 & 2s + 4s^2 & 1 + 3s + 9s^2 \\ 2 + 3s + 5s^2 & 2s + 4s^2 & 4 + 3s + 9s^2 \\ s & 2s & 3s \end{bmatrix} . G$$

where $q(s)$ is a system characteristic equation. Elements of a (3 x 3) constant matrix G must be determined. We can observe from W(s) that $Q\Gamma_i$ $i = 1, 2, 3$ are given by

$$Q\Gamma_1 = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 3 \\ 5 & 4 & 9 \end{bmatrix}, \quad Q\Gamma_2 = \begin{bmatrix} 2 & 0 & 4 \\ 3 & 2 & 3 \\ 5 & 4 & 9 \end{bmatrix}, \quad Q\Gamma_3 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}.$$

Since

$$\text{rank} \left(\sum_{i=1}^{3} \Gamma_i \right) = \text{rank} \left(\sum_{i=1}^{3} Q\Gamma_i \right) = \begin{bmatrix} 3 & 0 & 5 \\ 5 & 6 & 9 \\ 10 & 8 & 18 \end{bmatrix} = 3,$$

one solution of the matrix G is given by

$$G = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}. \tag{5.3}$$

In fact, any nonzero values can be assigned to the elements of the first column of the matrix G. For the matrix G of equation (5.3), the system w(s) is written by

$$W(s) = \frac{1}{q(s)} \begin{bmatrix} 2 + 6s + 18s^2 & 0 & 0 \\ 6 + 8s + 18s^2 & 0 & 0 \\ 6s & 0 & 0 \end{bmatrix}$$

In this section, the minimal input-output interaction problem was solved for a rather special form of the matrices G and F. A disadvantage of this approach is that the compensated system W(s) is regarded as a single input system, because zero vectors were assigned to $g_2, g_3, \ldots, g_r$. In the following section a nonsingularity assumption is placed on the matrix G in order to retain r input components of the system.

5.3 Selection of the Matrices G and F With Nonsingularity Assumption on G

Assume that the rank of the matrix G is r and that the system (1.1) to be compensated is output controllable. Then by Theorem 2.3 the system (1.3) is output controllable if and only if the matrices Γ_i i = 1, 2, ..., m are linearly independent. Thus, the matrix F must be chosen so as to satisfy the independence of Γ_i. Therefore, the objective of the minimal input-output interaction problem with the nonsingularity assumption on G is stated as follows: Choose a nonsingular matrix G and a matrix F in such a way that

(1) Γ_i $i = 1, 2, \ldots, m$ are linearly independent of each other; and

(2) the maximum number of zero elements are generated in $W(s)$.

Consider the selection of an appropriate matrix G, provided that the matrices Γ_i $i = 1, 2, \ldots, m$ are known. Let K be a set $\{1, 2, \ldots, m\}$ and let I_i be a proper subset of K, that is,

$$I_i \subset K.$$

There exist 2^m-1 different subsets of K. Denote these proper subsets by $I_1, I_2, \ldots, I_{2^m-1}$. Notice that the set consisting of I_i $i=1, 2, \ldots, 2^m-1$ and K is identified with the power set of K. Denote the number of elements in the set I_i by $|I_i|$ and define a set $K(I_i)$ as

$$K(I_i) = K - I_i = \{x \varepsilon K \mid x \notin I_i\}.$$

Furthermore, define $E_{K(I_i)}$ by

$$E_{K(I_i)} = \begin{bmatrix} \Gamma_{k_1} \\ \Gamma_{k_2} \\ \vdots \\ \Gamma_{k_i} \end{bmatrix}$$

where k_j are distinct elements of the set $K(I_i)$. Assume that $E_{K(\{0\})}$ is E_K.

Consider the equation

$$E_{K(I_i)}\, g_{I_i} = \underline{0} \tag{5.4}$$

In the footnote on page 43, it is stated that if rank $E_{K(I_i)} = h$ there exist $(r-h)$ independent solutions g_{I_i} which satisfy equation (5.4). First we find all possible nonzero solutions g_{I_i} for $i = 1, 2, \ldots, 2^m$. From the definition of $E_{K(I_i)}$, the solution g_{I_i} which satisfies equation (5.4) must meet the following equations simultaneously:

$$\begin{aligned} \Gamma_{k_1}\, g_{I_i} &= 0 \\ \Gamma_{k_2}\, g_{I_i} &= 0 \\ &\vdots \\ \Gamma_{k_i}\, g_{I_i} &= 0 \,. \end{aligned}$$

By referring to equation (2.12), if this solution g_{I_i} is assigned to the j-th column of the matrix G, the matrix W(s) generates zero on (k_1, j)-th, (k_2, j)-th, ..., (k_i, j)-th elements.

In order to find the maximum number of zero elements in W(s), we choose all nonzero solutions g_{I_i} satisfying equation (5.4) for each i. Then the solution g_{I_i} corresponding to the smallest value of $|I_i|$ is assigned to the first column of the matrix G. Next, the solution g_{I_i} corresponding to the second smallest value of $|I_i|$ is assigned to the second column of the matrix G. This selection is repeated until the r-th solution is obtained. However, we must always check the independency of the selected solutions g_{I_i}. That is, a solution which will be accepted to put in the matrix G must be independent of the column vectors being previously assigned in G.

When there are less than r nonzero independent solutions satisfying equation (5.4), these nonzero solutions are first assigned to the different columns of the matrix G and the rest of the elements in G are selected so that the matrix G becomes nonsingular.

Finally, the problem of choosing elements of the matrix F must be considered. As indicated before, the elements of the matrix F should be selected to satisfy the following two conditions:

(1) The matrices Γ_i $i = 1, 2, \ldots, m$ are linearly independent.

(2) The maximum number of zero elements are generated in W(s).

It appears to be difficult to specify such a matrix F satisfying the above two conditions. The general solution to this problem has not been obtained, but a conjecture will be proposed.

Let C(F) be a class of the matrices F which satisfy the first condition of independence. The class C(F) is not empty. For instance, a zero matrix F is qualified to be an element of C(F) as discussed in section 5.2. The selection of an element of C(F) which satisfies the second condition could require a cumbersome operation. A conjecture obtained from the observation of equation (5.4) concerning the selection of F is that the matrix F should be chosen so that for the smaller value of $|I_i|$ the rank $E_{K(I_i)}$ becomes as small as possible. Even the matrix E_K

can have a rank less than r by a proper choice of the matrix F.

In this section, the minimal imput-output interaction problem has been considered under the assumption that G is nonsingular. The selection of matrix G was extensively investigated, provided that Γ_i i = 1, 2, ..., m are known. However, the choice of F which actually determines the matrices Γ_i seems to require some other mathematical approaches.

CHAPTER VI

SOME PROPERTIES OF THE DECOUPLED SYSTEM

The significant advances in the theory of decoupling made in the last decade were described briefly in section 1.3. Recently, the scope of research on the theory of decoupling seems to be expanding gradually to cover broader topics. For example, efforts have been made to investigate such properties of the decoupled system as observability (cf. [3], [23]) and stability (cf. [18], [32], [35]). In this chapter, we consider three properties; stabilizability, parameter insensitivity, and functional reproducibility, and investigate the possible realization of these properties in the decoupled system. A mathematical interest will be directed toward the problem of selecting the compensators G and F such that the system can be decoupled and at the same time satisfy a specified property among three properties.

6.1 Stabilization of the Decoupled System

The purpose of this section is to investigate some conditions required in order to decouple and stabilize the system (1.3) simultaneously. Historically, Silverman [32] conjectured that the system (1.3) may not be decoupled and stabilized simultaneously. Later, Liu and Bergman [18] attempted to give a sufficient condition for decoupling and stabilization of the system (1.3). However, some counterexamples will be shown to exist for the sufficient condition. In this section, Liu and Bergman's condition will be examined and modified in order to complete the argument of stabilization of the decoupled system.

The system (1.1) is said to be stabilizable if there exists a constant feedback control $u = Fx$ such that the compensated system $\dot{x} = (A + BF)\,x$ is asymptotically stable. In other words, each eigenvalue of $A + BF$ has negative real part. We also have a feedforward compensator G in the system under consideration, but one may notice that the selection of the matrix G does not affect the system stability at all. Hence, the problem is to choose the appropriate matrices G and F so that the system (1.3) can be decoupled and also stabilized. A sufficient condition for decoupling and stabilization of the system (1.3) given by Liu and Bergman is stated as follows:

Theorem:

According to Liu and Bergman [18], a sufficient condition for the system (1.3) to be decoupled and stabilized is that (1) G^* in equation (4.19) is a nonsingular matrix; and (2) there exists a matrix F such that each of m row eigenvectors of (A+BF) is proportional to each of m rows of the output matrix C.

One of the counterexamples is given by the system for which the above conditions (1) and (2) are satisfied but $d_i > 1$ for some i. From $d_i > 1$, we obtain that $C_iB = 0$. Since the condition (2) states that $C_i(A+BF) = \lambda_iC_i$, the matrix Γ_i turns out to be given by

$$\begin{bmatrix} C_iB \\ \vdots \\ C_i(A+BF)^{d_i}B \\ C_i(A+BF)^{d_i+1}B \\ \vdots \\ C_i(A+BF)^{n-1}B \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ C_i(A+BF)^{d_i}B \\ C_i(A+BF)^{d_i+1}B \\ \vdots \\ C_i(A+BF)^{n-1}B \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ C_i\lambda_i^{d_i}B \\ C_i\lambda_i^{d_i+1}B \\ \vdots \\ C_i\lambda_i^{n-1}B \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \lambda_i^{d_i}C_iB \\ \lambda_i^{d_i+1}C_iB \\ \vdots \\ \lambda_i^{n-1}C_iB \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

The matrix Γ_i has rank zero. Then the decoupling condition of Theorem 4.3 is ~~not~~ satisfied. Thus, Liu and Bergman's condition is not sufficient.

A modification of Liu and Bergman's condition is given in the following theorem.

Theorem 6.1

A sufficient condition for decoupling and stabilization of the system (1.3) is that (1) CB is nonsingular; and (2) there exists a matrix F such that each of m row eigenvectors of (A+BF) is proportional to each of m rows of the output matrix C.

Proof:

Since CB is nonsingular, G* becomes nonsingular and $C_iB \neq 0$ for each i. In fact, G* = CB. Then the matrix Γ_i is written in the form

$$\Gamma_i = \begin{bmatrix} C_iB \\ C_i(A+BF)B \\ \vdots \\ C_i(A+BF)^{n-1}B \end{bmatrix} = \begin{bmatrix} C_iB \\ C_i\lambda_iB \\ \vdots \\ C_i\lambda_i^{n-1}B \end{bmatrix} = \begin{bmatrix} C_iB \\ \lambda_iC_iB \\ \vdots \\ \lambda_i^{n-1}C_iB \end{bmatrix} \quad \text{for } i=1,2,\ldots,m$$

Obviously, Γ_i has rank one for each i. By Theorem 4.3, the system (1.3) is decoupled.

We will examine the stabilizability of the system (1.3). The condition (2) requires that

$$C_i(A + BF) = \lambda_iC_i \qquad \text{for } i = 1, 2, \ldots, m \tag{6.1}$$

The nm elements in F must be determined so that equation (6.1) is satisfied. Since there exist nm unknowns (all elements in F) and nm linear equations, each element in F can be solved as a function of λ_i. The eigenvalues λ_i can be arbitrarily assigned so that the system (1.3) becomes stable. If some of λ_i are complex, the conjugate pairs must exist in order to guarantee real solutions for elements of F.

Q.E.D.

Theorem 6.1 is more generalized in the following theorem:

Theorem 6.2

A sufficient condition for decoupling and stabilization of the system (1.3) is that (1) G^* is nonsingular; and (2) there exists a matrix F satisfying $C_i(A+BF)^{d_i+1} = \lambda_i C_i$ for i=1, 2, ..., m.

Proof:

Consider the row vectors of Γ_i. From the definition of d_i, $C_i(A+BF)^j B = 0$ for $j = 0, 1, \ldots, d_i-1$ and $C_i(A+BF)^{d_i}B = C_i A^{d_i} B \neq 0$. For $k = 1, 2, \ldots, n-d_i-1$,

$$C_i(A+BF)^{d_i+k} = C_i(A+BF)^{d_i+1}(A+BF)^{k-1}B$$

$$= \lambda_i C_i (A+BF)^{k-1}B$$

$$= \begin{cases} 0 & \text{if } k-1 < d_i \\ \lambda_i C_i A^{d_i} B & \text{if } k-1 = d_i \\ \lambda_i^2 C_i (A+BF)^{k-2-d_i} B & \text{if } k-1 > d_i \end{cases}$$

The third solution is further reduced to 0 or $\lambda_i^3 C_i (A+BF)^{k-3-2d_i}$ depending on the value of the exponent of (A+BF). The operation is repeated until the third solution finally gets to zero or the form $\lambda_i^j C_i A^{d_i} B$, where j is some integer. Then rank Γ_i becomes one for each i. By Theorem 4.3, the system (1.3) is decoupled. Eigenvalues are arbitrarily assigned in a similar way as discussed in the proof of Theorem 6.1.

Q.E.D.

Based on Liu and Bergman's result, a sufficient condition for decoupling and stabilization of the system (1.3) was examined and generalized. The critical mistake made in the sufficient condition by Liu and Bergman is the nonsingularity assumption of G*. As stated in Theorem 6.1, the stronger restriction is required to ensure sufficiency. That is, instead of the nonsingularity assumption on G*, the nonsingularity of the matrix CB is required. The generalization of Liu and Bergman's result is also given by Theorem 6.2.

6.2 Parameter Insensitivity of the Decoupled System

Sensitivity analysis plays an important role in system analysis and synthesis (cf. [21], [17]). Parameter sensitivity is expressed in terms of a partial derivative of a system characteristic with respect to a change in a system parameter. The system characteristic is taken to be some measure or response of the system, and a typical system parameter may be a coefficient of the system equations. In this section, the system characteristic desired will be the decoupling property, and the system parameter considered will be a differential change of an element of the matrix A.

The system transfer function matrix of the decoupled system (1.3) is given by equation (1.4), where F and G have been selected so that W(s) has a diagonal form. The investigation will develop some insensitivity conditions for the decoupled system in terms of a differential change of an element of the matrix A. This differential change might influence the decoupling property of the system (1.3) in a complicated manner, since the matrix A operates as an inverse in the system transfer function matrix W(s). The following mathematical development is based partly on Gass's work [10] in sensitivity analysis of linear programming.

Assume that an (i,j)-th element of matrix A changes by Δa_{ij}. Let E_{ij} be a matrix whose elements are zero except one at the (i,j)-th position, i. e.,

$$E_{ij} = \begin{array}{cc} \begin{array}{c} \text{j-th col} \\ \downarrow \end{array} & \\ \begin{bmatrix} 0 & \dots & 0 & \dots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \dots & 1 & \dots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \dots & 0 & \dots & 0 \end{bmatrix} & \leftarrow \text{i-th row} \end{array}$$

The transfer function matrix of the system with the change of Δa_{ij} is described by

$$\bar{W}(s) = C(sI-(A + \Delta a_{ij}E_{ij}) - BF)^{-1}BG$$

$$= C(sI-A-BF + (-\Delta a_{ij}E_{ij}))^{-1}BG \tag{6.2}$$

Defining $P = (sI - A - BF)$, equation (6.2) is expressed as

$$\bar{W}(s) = C(P + (-\Delta a_{ij}E_{ij}))^{-1}BG$$

$$= C(P(I + (-P^{-1} \Delta a_{ij}E_{ij})))^{-1}BG$$

$$= C(I + (-P^{-1} \Delta a_{ij}E_{ij}))^{-1}P^{-1}BG \tag{6.3}$$

An investigation is further developed for one of the inverse terms in equation (6.3).

$$
(I + (-P^{-1}\Delta a_{ij}E_{ij}))^{-1} = I + \begin{array}{c} \text{j-th col} \\ \begin{bmatrix} 0 & \cdots & -p'_{1i}\,\Delta a_{ij} & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \cdots & -p'_{ji}\,\Delta a_{ij} & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \cdots & -p'_{ni}\,\Delta a_{ij} & \cdots & 0 \end{bmatrix}^{-1} \end{array} \qquad (6.4)
$$

where $P^{-1} = [p'_{ij}]$. Equation (6.4) is reduced to

$$
(I + (-P^{-1}.\ \Delta a_{ij}E_{ij}))^{-1} = \begin{bmatrix} 1 & \cdots & -p'_{1i}\,\Delta a_{ij} & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \cdots & 1-p'_{ji}\,\Delta a_{ij} & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \cdots & -p'_{ni}\,\Delta a_{ij} & \cdots & 1 \end{bmatrix}^{-1}
$$

$$
= \begin{bmatrix} 1 & \cdots & p'_{1i}\,\Delta a_{ij}/(1-p'_{ji}\Delta a_{ij}) & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \cdots & 1/(1-p'_{ji}\Delta a_{ij}) & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \cdots & p'_{ni}\,\Delta a_{ij}/(1-p'_{ji}\Delta a_{ij}) & \cdots & 1 \end{bmatrix}
$$

$$
= I + \begin{bmatrix} 0 & \cdots & p'_{1i}\,\Delta a_{ij}/(1-p'_{ji}\,\Delta a_{ij}) & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & & p'_{ji}\,\Delta a_{ij}/(1-p'_{ji}\,\Delta a_{ij}) & \cdots & 0 \\ \vdots & & & & \vdots \\ 0 & \cdots & p'_{ni}\,\Delta a_{ij}/(1-p'_{ji}\,\Delta a_{ij}) & \cdots & 0 \end{bmatrix}
$$

$$= I + \Delta P \tag{6.5}$$

where

$$\Delta P = \begin{bmatrix} 0 & \cdots & p'_{1i}\ \Delta a_{ij}/(1-p'_{ji}\ \Delta a_{ij}) & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \cdots & p'_{ji}\ \Delta a_{ij}/(1-p'_{ji}\ \Delta a_{ij}) & \cdots & 0 \\ \vdots & & \vdots & & \vdots \\ 0 & \cdots & p'_{ni}\ \Delta a_{ij}/(1-p'_{ji}\ \Delta a_{ij}) & \cdots & 0 \end{bmatrix}$$

The substitution of equation (6.5) into equation (6.3) yields

$$\begin{aligned} \bar{W}(s) &= C(I + \Delta P)\ P^{-1}BG \\ &= C\ P^{-1}\ BG + C\Delta P\ P^{-1}\ BG \\ &= C(sI - A - BF)^{-1}BG + C\Delta P(sI - A - BF)^{-1}BG \end{aligned} \tag{6.6}$$

Let us now consider the condition of parameter insensitivity. Since G and F have been chosen so that the system W(s) is decoupled, the first term of equation (6.6) is diagonalized. Therefore, from equation (6.6) the decoupling property of the system $\bar{W}(s)$ is retained if and only if the second term vanishes or has a diagonal form. Since the system W(s) is decoupled, the matrix $(sI - A - BF)^{-1}BG$ can not be a zero matrix. Then the second term can be diagonalized if and only if

$$p'_{1i} = p'_{2i} = \cdots = p'_{(j-1)i} = p'_{(j+1)i} = \cdots = p'_{ni} = 0 \text{ and } p'_{ji} \neq 0.$$

The second term vanishes if and only if

$$p'_{1i} = p'_{2i} = \ . \ . \ . = p'_{ni} = 0.$$

However, this is impossible, because $p'_{ki} = 0 \quad k = 1, 2, \ . \ . \ ., n$ imply that the matrix $(sI - A - BF)^{-1}$ has a zero column. Then, the immediate conclusion is stated as follows:

<u>Theorem 6.3</u>

The decoupled system (1.3) is parameter insensitive with respect to the (i,j)-th element of the matrix A if and only if

$$p'_{1i} = p'_{2i} = \cdots = p'_{(j-1)i} = p'_{(j+1)i} = \cdots = p'_{ni} = 0 \text{ and } p'_{ji} \neq 0$$

where $[p'_{ij}] = (sI - A - BF)^{-1}$, and 0 represents zero rational function of s.

From Theorem 6.3, which is the main result of this section, we observe that the elements $p'_{ki} \quad k = 1, 2, \ldots, n$ do not depend on the matrix G. Therefore, in designing the decoupled system which is also parameter insensitive with respect to the (i,j)-th element of the matrix A, we should choose the matrix G which satisfies the decoupling condition and should choose the matrix F which satisfies both the decoupling condition and Theorem 6.3. However, the existence of such a matrix F has not been discussed in this section.

6.3 Functional Reproducibility of the Decoupled System

Brockett and Mesarovic [2] presented the concept of functional reproducibility concerning the control of the output of a dynamical system over a time interval. The objective of this section is to show that the decoupled system (1.3) is functionally reproducible.

The definition and an existing result of functional reproducibility for the system (1.1) will be presented by summarizing Brockett and Mesarovic [2]. Denote the solution of the system (1.1) which results from a forcing function $u(t)$ being applied at $t = 0$ with initial state β by $C\,x(\beta, u, t)$. The least restrictive concept of the functional reproducibility is explained as follows: Let T be a set of time at which the behavior of the system is defined, and let $\tau \in T$. Denote a homogeneous response by $y'(t) = C\,x(\beta, 0, t)$. Define a desired response $y(t)$ which is sufficiently close to $y'(t)$. Then if there exists a u such that a response $C\,x(\beta, u, t)$ agrees with the $y(t)$ at any time in $[0,\tau]$, the system will be said to be functionally reproducible. Here the word "close" will be interpreted in the sense of norm. Assume that α is a k-times differentiable function of time. Two types of norms will be used. Define $\|\alpha\|$ as

$$\|\alpha\| = \sup_{t\in[0,\tau]} |\alpha(t)|$$

where $|\alpha(t)|$ denotes the sum of the absolute values of the elements.

Define

$$\|\alpha\|_p = \max_{0 \le k \le p} \| \alpha^{(k)} \|$$

where $\alpha^{(k)}$ denotes the k-th derivative of $\alpha(t)$. The functional reproducibility is now defined mathematically as follows:

Definition

The homogeneous response from an initial state β is said to be functionally reproducible if, for any $\eta > 0$ and finite $\tau > 0$, there exists a $\delta(\eta,\tau) > 0$ such that corresponding to each y for which

$$\|y - C\, x(\beta,\, 0,\, t)\|_n < \delta(\eta,\tau),$$

there is a u having the properties: $\|u\| < \eta$ and $C\, x(\beta,\, u,\, t) = y(t)$ for all values of t in the interval $[0,\tau]$.

The following theorem shows a necessary and sufficient condition for the system to be functionally reproducible [2].

Theorem 6.4

For the system (1.1), all homogeneous responses of the system are functionally reproducible if and only if the following $(mn \times r(2n-1))$ matrix

$$
M_n = \begin{bmatrix}
CB & CAB & CA^2B & \dots & CA^{n-1}B & CA^nB & \dots & CA^{2n-2}B \\
0 & CB & CAB & \dots & CA^{n-2}B & CA^{n-1}B & \dots & CA^{2n-3}B \\
0 & 0 & CB & \dots & CA^{n-3}B & CA^{n-2}B & \dots & CA^{2n-4}B \\
\vdots & & & & \vdots & & & \vdots \\
0 & 0 & 0 & \dots & CB & CAB & \dots & CA^{n-1}B
\end{bmatrix}
$$

is of rank mn.

We shall now show that the decoupled system (1.3) is functionally reproducible. By Theorem 6.4 all homogeneous responses of system (1.3) are functionally reproducible if and only if

$$
M_n = \begin{bmatrix}
CBG & C\bar{\bar{A}}BG & C\bar{\bar{A}}^2BG & \dots & C\bar{\bar{A}}^{n-1}BG & C\bar{\bar{A}}^nBG & \dots & C\bar{\bar{A}}^{2n-2}BG \\
0 & CBG & C\bar{\bar{A}}BG & \dots & C\bar{\bar{A}}^{n-2}BG & C\bar{\bar{A}}^{n-1}BG & \dots & C\bar{\bar{A}}^{2n-3}BG \\
0 & 0 & CBG & \dots & C\bar{\bar{A}}^{n-3}BG & C\bar{\bar{A}}^{n-2}BG & \dots & C\bar{\bar{A}}^{2n-4}BG \\
\vdots & & \vdots & & & \vdots & & \\
0 & 0 & 0 & \dots & CBG & C\bar{\bar{A}}BG & \dots & C\bar{\bar{A}}^{n-1}BG
\end{bmatrix} \tag{6.7}
$$

is of rank mn, where $\bar{\bar{A}} = (A+BF)$.

A main contribution of the decoupling property of the system (1.3) to the functional reproducibility is given in the following theorem.

<u>Theorem 6.5</u>

If system (1.3) is decoupled, then all homogeneous responses of the system are functionally reproducible.

Proof:

Since the system (1.3) is decoupled, the following three conditions are satisfied.

(1) There exist the matrices G and F for which Γ_i is described by

$$\Gamma_i = \begin{bmatrix} C_i(A+BF)^0 B \\ C_i(A+BF)^1 B \\ \vdots \\ C_i(A+BF)^{n-1} B \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ C_i(A+BF)^{d_i} B \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ C_i A^{d_i} B \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

for each i.

(2)

$$G^* = \begin{bmatrix} C_1 A^{d_1} B \\ C_2 A^{d_2} B \\ \vdots \\ C_m A^{d_m} B \end{bmatrix} \quad \text{is of rank } m.$$

(3) G is a nonsingular matrix.

It will be seen from the above three conditions that

$$\begin{bmatrix} C_i \quad BG \\ C_i(A+BF)^1 BG \\ \vdots \\ C_i(A+BF)^{n-1} BG \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ C_i A^{d_i} BG \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{6.8}$$

for each i, and that

$$\operatorname{rank} \begin{bmatrix} C_1A^{d_1}BG \\ C_2A^{d_2}BG \\ \vdots \\ C_mA^{d_m}BG \end{bmatrix} = m. \tag{6.9}$$

To prove Theorem 6.5 it suffices to show that the matrix in equation (6.7) has rank mn when equation (6.8) and equation (6.9) hold. Since the rank of a matrix does not change by the elementary transformation, we define a matrix $M_n{}'$ which is obtained by interchanging rows of M_n in equation (6.7) as follows:

$$M_n{}' = \left[\begin{array}{cccccccc}
C_1BG & C_1\bar{\bar{A}}BG & C_1\bar{\bar{A}}^2BG & \cdots & C_1\bar{\bar{A}}^{n-1}BG & C_1\bar{\bar{A}}^nBG & \cdots & C_1\bar{\bar{A}}^{2n-2}BG \\
0 & C_1BG & C_1\bar{\bar{A}}BG & \cdots & C_1\bar{\bar{A}}^{n-2}BG & C_1\bar{\bar{A}}^{n-1}BG & \cdots & C_1\bar{\bar{A}}^{2n-3}BG \\
0 & 0 & C_1BG & \cdots & C_1\bar{\bar{A}}^{n-3}BG & C_1\bar{\bar{A}}^{n-2}BG & \cdots & C_1\bar{\bar{A}}^{2n-4}BG \\
\vdots & & & & \vdots & & & \vdots \\
0 & 0 & 0 & \cdots & C_1BG & C_1\bar{\bar{A}}BG & \cdots & C_1\bar{\bar{A}}^{n-1}BG \\
\hline
C_2BG & C_2\bar{\bar{A}}BG & C_2\bar{\bar{A}}^2BG & \cdots & C_2\bar{\bar{A}}^{n-1}BG & C_2\bar{\bar{A}}^nBG & \cdots & C_2\bar{\bar{A}}^{2n-2}BG \\
\vdots & & & & \vdots & & & \vdots \\
0 & 0 & 0 & & C_2BG & C_2\bar{\bar{A}}BG & \cdots & C_2\bar{\bar{A}}^{n-1}BG \\
\hline
& \cdots & & & \cdots & & & \cdots \\
\hline
C_mBG & C_m\bar{\bar{A}}BG & C_m\bar{\bar{A}}^2BG & \cdots & C_m\bar{\bar{A}}^{n-1}BG & C_m\bar{\bar{A}}^nBG & \cdots & C_m\bar{\bar{A}}^{2n-2}BG \\
\vdots & & & & \vdots & & & \vdots \\
0 & 0 & 0 & \cdots & C_mBG & C_m\bar{\bar{A}}BG & \cdots & C_m\bar{\bar{A}}^{n-1}BG
\end{array}\right]$$

where C_i is the i-th row of matrix C. From the definition of d_i, $M_n{}'$ is reduced to

$$
M_n{}' = \left[\begin{array}{cccccccccc}
0 \cdots 0 & C_1A^{d_1}BG & 0 & \cdots & & & & & & 0 \\
0 \cdots 0 & 0 & C_1A^{d_1}BG & 0 & \cdots & & & & & 0 \\
\vdots & & \vdots & & & & & \vdots & & \\
0 \cdots 0 & 0 & 0 & \cdots & & 0 & C_1A^{d_1}BG & 0 & \cdots & 0 \\
\hline
0 & C_2A^{d_2}BG & 0 & \cdots & & & & & & 0 \\
0 & 0 & C_2A^{d_2}BG & 0 & \cdots & & & & & 0 \\
\vdots & & \vdots & & & & & \vdots & & \\
0 & 0 & 0 & \cdots & 0 & C_2A^{d_2}BG & 0 & \cdots & & 0 \\
\hline
\cdots & & & \cdots & & & & \cdots & & \\
\hline
0 & \cdots & 0 & C_mA^{d_m}BG & 0 & \cdots & & & & 0 \\
0 & \cdots & & & 0 & C_mA^{d_m}BG & 0 & \cdots & & 0 \\
\vdots & & \vdots & & & & & \vdots & & \\
0 & \cdots & & & & & & 0 & C_mA^{d_m}BG & 0
\end{array}\right]
$$

By equation (6.9), matrix $M_n{}'$ has rank mn. Therefore,

$$\operatorname{rank} M_n = \operatorname{rank} M_n{}' = mn$$

Then the system (1.3) is functionally reproducible.

Q.E.D.

The functional reproducibility of the decoupled system was proved in this section. In some sense, the functional reproducibility can be interpreted as a restricted property of the system output controllability. Therefore, it was clarified in this section that the decoupled system is functionally reproducible as well as output controllable.

CHAPTER VII

CONCLUSIONS AND DIRECTIONS FOR FURTHER WORK

7.1 Conclusions

The dissertation focuses on the investigation of noninteraction properties of linear time-invariant multivariable control systems. Specifically, the objective was to obtain solutions to the selective output invariance problem, the decoupling problem, and the minimal input-output interaction problem. The primary feature which distinguishes this work from the efforts cited in the references is the introduction of the notion of the row coefficient matrix (RCM). The use of the RCM itself or its components in the system representation has showed the advantage of providing an intuitive explanation of the solution procedure appropriate to noninteraction problems.

The selective output invariance property has been considered as fundamental in decoupling and minimal input-output interaction, and the selective output invariance property could probably serve also as the basis for characterizing the degree of noninteraction properties in other kinds of noninteraction problems as well. A sufficient condition for selective output invariance was proposed under the assumption that the number of output components is larger than or equal to the number of input components. Since only sufficiency was proved, the system which satisfies this sufficient condition exhibits the i-th output invariance with respect to the j-th input, but it may also exhibit some undesirable side effects. In other words, the compensation by matrices G and F specified by this sufficient condition may result in the invariance of output elements for which the property of invariance may if fact not be desired. To overcome this difficulty, the necessary condition of the selective output invariance should be investigated.

One of the main results of the dissertation has been the development of a methodology for the decoupling problem. A smooth transition from the discussion of the selective output invariance problem to one concerned with the decoupling problem was achieved by virtue of introduction of RCM's. A necessary and sufficient condition was proved and the relationship between this condition and Falb and Wolovich's result [7] was clarified. The matrices Γ_i, which are components of the RCM's, played an important role in providing a simple mathematical proof of the necessary and sufficient condition. Consequently, less mathematical knowledge is required for reader to understand this proof than for understanding the proofs contained in the various other expositions cited in this dissertation. A part of the proof (Lemma 4.1) can be simplified if mathematical lattice theory is introduced to show some partial ordering among matrices $E_{K(I_i)}$ for $i = 1, 2, \ldots, 2^m-1$. The notion of selective decoupling was also introduced as a theoretical extension of the decoupling problem.

The minimal input-output interaction problem was solved for the two special cases: One is the system (1.3) with the nonsingularity assumption on G and the other is the system (1.3) without such an assumption. The methodology seems to employ a trial-and-error approach, but some ideas for improvement will be proposed in the following section. It would be interesting to investigate the necessary and sufficient condition for the minimal input-output interaction of the system (1.3), possibly along the lines of Sato and Lopresti [30].

The stabilization, parameter insensitivity, and functional reproducibility of the decoupled system (1.3) were considered to be an extension of the thesis topic. As for the stabilization of the decoupled system, a mistake in Liu and Bergman's work [18] was identified and a modification suggested. A generalization of the sufficient condition was also proposed. For the parameter insensitivity, only a differential change of an element of the matrix A was considered, and a sufficient condition was derived. As an immediate extension of this research, one could consider sensitivity analysis with respect to the differential change of the matrix B and the matrix C. Similarly, the study of the insensitivity property with respect to a change of more than one component of the matrix A would be interesting. As to the functional reproducibility of the decoupled system (1.3), it is of interest that the decoupled system already exhibits the functional reproducibility property which is considered to be a restricted concept of the output controllability.

7.2 Direction for Further Work

Although some recommendations for further work were indicated in the previous section, there are several other directions that also appear to merit further investigation.

As pointed out in section 7.1, the minimal input-output interaction problem was solved by a rather heuristic approach, and was originally considered to be an extension of the decoupling problem. However, the nature of the problem has turned out to be more complicated than would be the case if it were merely a direct extension of the decoupling problem. For instance, methodologies similar to those used for selective decoupling and partial decoupling [30] were applied to solve the problem, but they proved to be inadequate to produce enough information about the portion of the system which was not decoupled to permit further analysis. Thus, the minimal input-output interaction problem can not be adequately analyzed as an extension of the decoupling problem. A proposed direction for further work then, would be to find methodologies appropriate to the minimal input-output interaction problem as well as to elaborate on the methodologies used for selective and partial decoupling. The establishment of a condition which characterizes the minimal input-output interaction - for instance, the range of the number of zero elements in the system transfer function matrix $W(s)$ - could be a promising approach.

As for the selective output invariance problem, that problem should be extended to cover the case in which the number of input components is less than the number of output components (i. e., $r < m$).

For this case, a different selection of matrices G and F (different from those discussed in section 3.2) should be considered. The derivation of a necessary and sufficient condition is also desirable.

An output controllability criterion was described in terms of the RCM's of the system (1.3). It can be shown that some traditional criteria concerning state controllability, observability, selective state controllability and selective observability can be similarly developed in terms of the RCM's.

Finally, as a general comment that has been made in a number of the works cited as references in this dissertation, the scope of the noninteraction problems can be extended to the problem of studying the properties when the nonsingularity assumption on G is relaxed, and to the problem of searching the conditions of noninteraction properties when the feedforward compensator G and output feedback compensator F are available, and also to the problem of investigating the noninteraction properties when only the feedforward compensator G is available.

APPENDICES

APPENDIX A

The quantity d_i is defined by

$$d_i = \min \{ j : C_i A^j B \neq 0 \quad \text{for } j = 0, 1, 2, \ldots, n-1\}$$

or

$$d_i = n-1 \quad \text{if} \quad C_i A^j B = 0 \text{ for all } j, \tag{A.1}$$

for i = 1, 2, ..., m. In this appendix, we show that

$$C_i(A+BF)^j B = \begin{cases} C_i A^j B & \text{for } j = 0, 1, \ldots, d_i \\ C_i A^{d_i}(A + BF)^{j-d_i} B & \text{for } j = d_i+1, \ldots, n-1 \end{cases} \tag{A.2}$$

The following lemma proves the equality of equation (A.2). Although the result of the lemma is presented in Falb and Wolovich [7] without proof, a complete proof by mathematical induction is now attached.

<u>Lemma A.1</u>

$$C_i(A+BF)^j = C_i A^j \quad \text{for} \quad j = 0, 1, \ldots, d_i \tag{A.3}$$

Proof:

For $j = 0$

$$C_i(A+BF)^0 = C_i\,. \tag{A.4}$$

Then equation (A.3) holds. For $j = 1$,

$$C_i(A+BF) = C_iA + C_iBF.$$

From the definition of d_i in equation (A.1), the second term turns out to be zero. Then equation (A.3) holds. Assume that equation (A.3) holds for $j = k$ ($\leq d_i$), that is, assume that

$$C_i(A+BF)^k = C_iA^k. \tag{A.5}$$

Then

$$\begin{aligned} C_i(A+BF)^{k+1} &= C_i(A+BF)^k(A+BF) \\ &= C_iA^k(A+BF) \\ &= C_iA^{k+1} + C_iA^kBF. \end{aligned}$$

From the definition of d_i, the second term becomes zero for $k \leq (d_i-1)$. Then

$$C_i(A+BF)^{k+1} = C_iA^{k+1}.$$

It follows that if equation (A.5) is true (i. e., if the equality holds for $j = k$), then equation (A.6) is true (i. e., the equality holds for $j = k+1$). But by equation (A.4) the equality holds for $k = 0$. Hence, it is true for $j = 0+1 = 1$, and $j = 1+1 = 2$, etc., and thus for $j = 0, 1, \ldots, d_i$.

Q.E.D.

Multiplying both sides of equation (A.3) by matrix B from the right, we obtain

$$C_i(A+BF)^jB = C_iA^jB \qquad \text{for } j = 0, 1, \ldots, d_i$$

For $j = d_i+1, \ldots, n-1$

$$C_i(A+BF)^jB = C_i(A+BF)^{d_i}(A+BF)^{j-d_i}B$$

$$= C_iA^{d_i}(A+BF)^{j-d_i}B$$

Then equation (A.2) was proved.

APPENDIX B

<u>Proposition</u>

For (n x m) matrices $A_1, A_2, \ldots, A_m$, there exists a vector x such that $A_1x, A_2x, \ldots, A_mx$ become linearly independent if $\sum_{i=1}^{m} A_i$ is of rank m and the elements of x are all nonzero.

Proof:

Let $A_1 = [a_{ij}^{1}]$, $A_2 = [a_{ij}^{2}]$, ..., $A_m = [a_{ij}^{m}]$. Construct a linear combination of A_ix $i = 1, 2, \ldots, m$ and set it equal to zero, i.e.,

$$c_1A_1x + c_2A_2x + \ldots + c_mA_mx = 0. \tag{B.1}$$

Denoting the column vector x by $x = \{x_1, x_2, \ldots, x_m\}'$, equation (B.1) is written in the form of

$$\sum_{j=1}^{m} c_j \left(\sum_{k=1}^{m} a_{ij}^{k}\right) x_j = 0 \quad \text{for } i = 1, 2, \ldots, m \tag{B.2}$$

In equation (B.2), if $\sum_{i=1}^{m} A_i$ is of rank m, we obtain that

$$c_1x_1 = c_2x_2 = \ldots = c_mx_m = 0.$$

Since $x_i \neq 0$ for $i = 1, 2, \ldots, m$,

$$c_1 = c_2 = \ldots = c_m = 0$$

Therefore, A_1x, A_2x, ..., A_mx are linearly independent.

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